

# A Story of Ratios<sup>®</sup>

## Eureka Math<sup>™</sup> Grade 6, Module 5

### Student File\_A

*Contains copy-ready classwork and homework*

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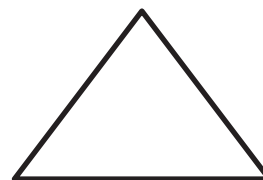
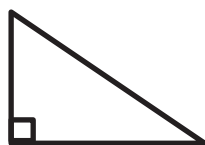
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## Lesson 1: The Area of Parallelograms Through Rectangle Facts

### Classwork

#### Opening Exercise

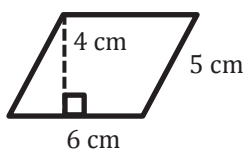
Name each shape.



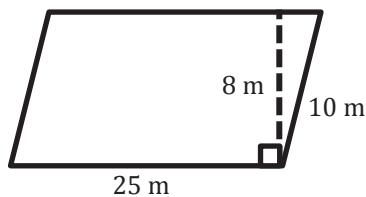
#### Exercises

1. Find the area of each parallelogram below. Note that the figures are not drawn to scale.

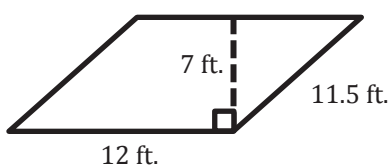
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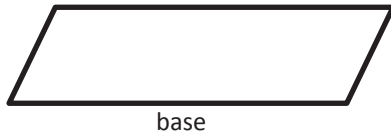


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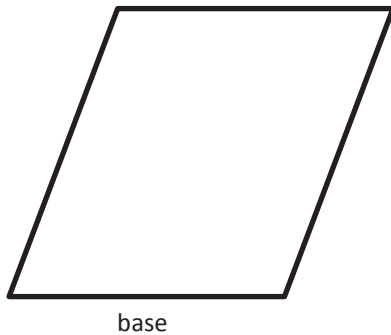


2. Draw and label the height of each parallelogram. Use the correct mathematical tool to measure (in inches) the base and height, and calculate the area of each parallelogram.

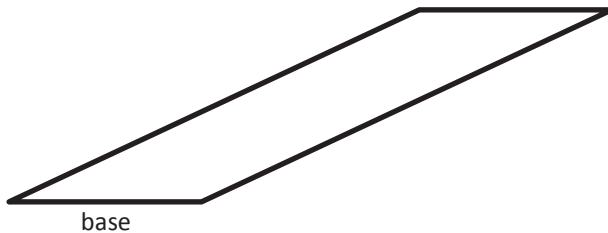
a.



b.



c.



3. If the area of a parallelogram is  $\frac{35}{42} \text{ cm}^2$  and the height is  $\frac{1}{7} \text{ cm}$ , write an equation that relates the height, base, and area of the parallelogram. Solve the equation.

### Lesson Summary

The formula to calculate the area of a parallelogram is  $A = bh$ , where  $b$  represents the base and  $h$  represents the height of the parallelogram.

The height of a parallelogram is the line segment perpendicular to the base. The height is usually drawn from a vertex that is opposite the base.

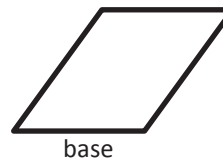
### Problem Set

Draw and label the height of each parallelogram.

1.

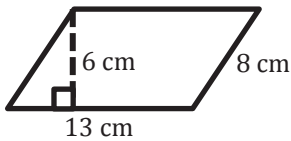


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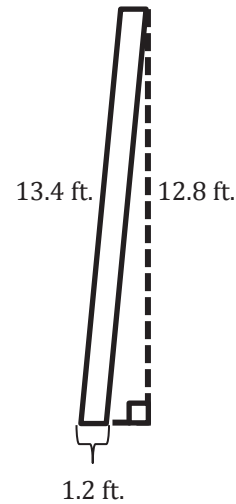


Calculate the area of each parallelogram. The figures are not drawn to scale.

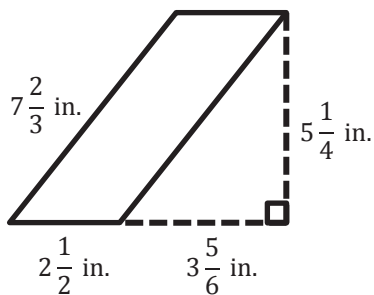
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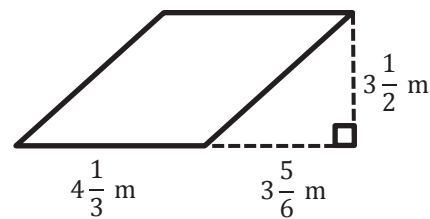
4.



5.



6.

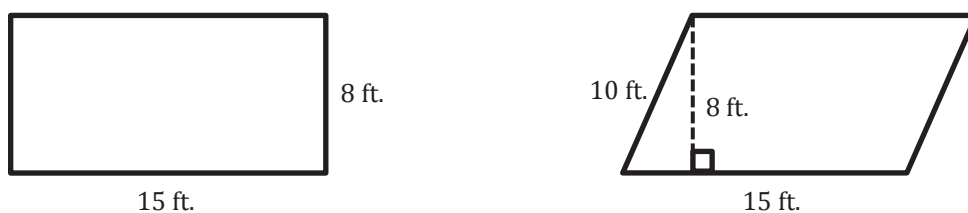


7. Brittany and Sid were both asked to draw the height of a parallelogram. Their answers are below.



Are both Brittany and Sid correct? If not, who is correct? Explain your answer.

8. Do the rectangle and parallelogram below have the same area? Explain why or why not.



9. A parallelogram has an area of  $20.3 \text{ cm}^2$  and a base of  $2.5 \text{ cm}$ . Write an equation that relates the area to the base and height,  $h$ . Solve the equation to determine the height of the parallelogram.

## Lesson 2: The Area of Right Triangles

### Classwork

#### Exploratory Challenge

- a. Use the shapes labeled with an X to predict the formula needed to calculate the area of a right triangle. Explain your prediction.

Formula for the area of right triangles: \_\_\_\_\_

Area of the given triangle: \_\_\_\_\_

- b. Use the shapes labeled with a Y to determine if the formula you discovered in part (a) is correct.

Does your area formula for triangle Y match the formula you got for triangle X?

If so, do you believe you have the correct formula needed to calculate the area of a right triangle? Why or why not?

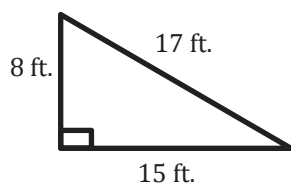
If not, which formula do you think is correct? Why?

Area of the given triangle: \_\_\_\_\_

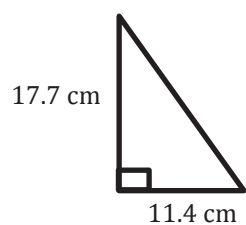
## Exercises

Calculate the area of each triangle below. Each figure is not drawn to scale.

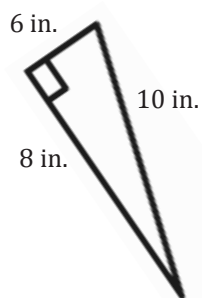
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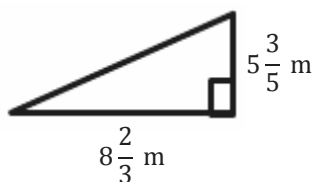
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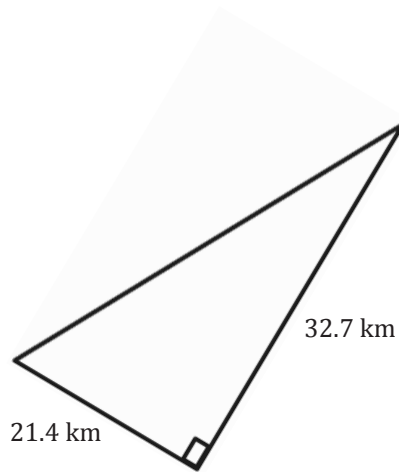
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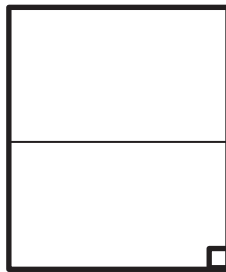


5.

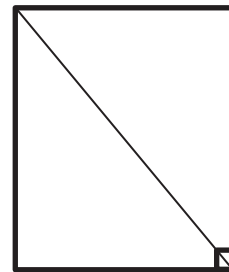


6. Mr. Jones told his students they each need half of a piece of paper. Calvin cut his piece of paper horizontally, and Matthew cut his piece of paper diagonally. Which student has the larger area on his half piece of paper? Explain.

Calvin's Paper



Matthew's Paper



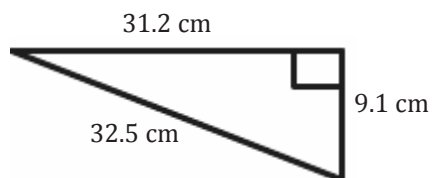
7. Ben requested that the rectangular stage be split into two equal sections for the upcoming school play. The only instruction he gave was that he needed the area of each section to be half of the original size. If Ben wants the stage to be split into two right triangles, did he provide enough information? Why or why not?
8. If the area of a right triangle is 6.22 sq. in. and its base is 3.11 in., write an equation that relates the area to the height,  $h$ , and the base. Solve the equation to determine the height.



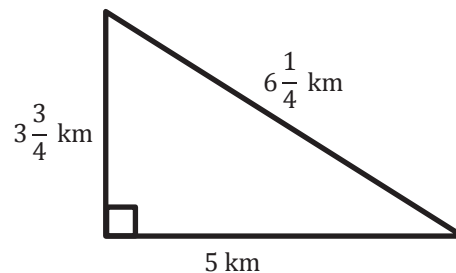
## Problem Set

Calculate the area of each right triangle below. Note that the figures are not drawn to scale.

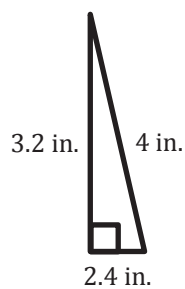
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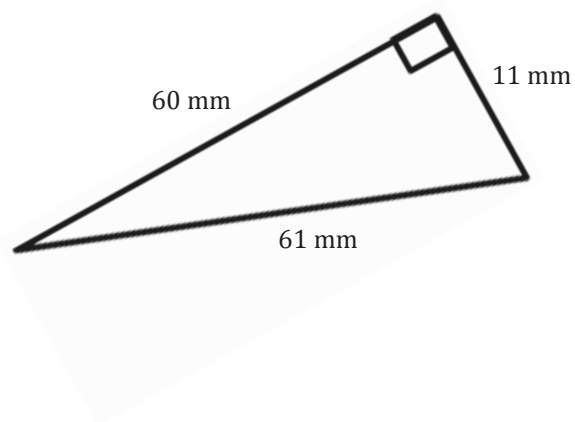
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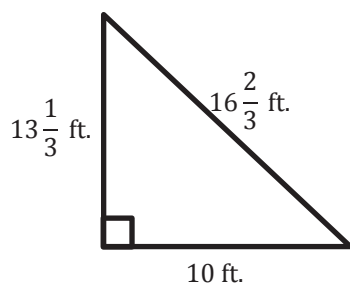
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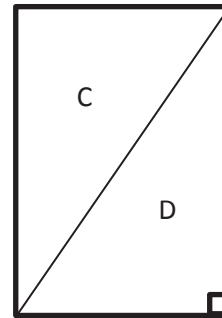
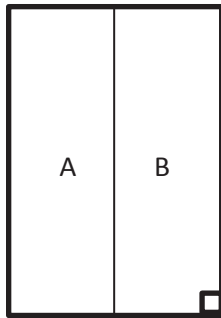
4.



5.



6. Elania has two congruent rugs at her house. She cut one vertically down the middle, and she cut diagonally through the other one.



After making the cuts, which rug (labeled A, B, C, or D) has the larger area? Explain.

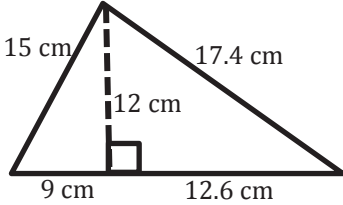
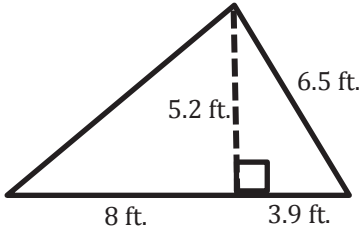
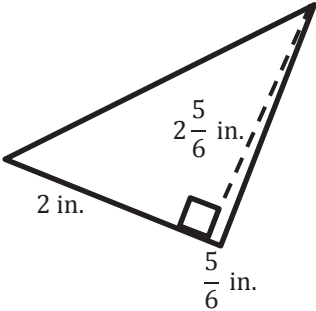
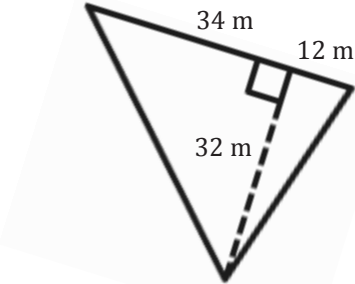
7. Give the dimensions of a right triangle and a parallelogram with the same area. Explain how you know.
8. If the area of a right triangle is  $\frac{9}{16}$  sq. ft. and the height is  $\frac{3}{4}$  ft., write an equation that relates the area to the base,  $b$ , and the height. Solve the equation to determine the base.

## Lesson 3: The Area of Acute Triangles Using Height and Base

### Classwork

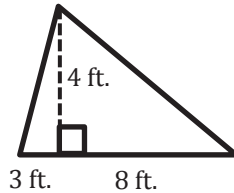
#### Exercises

1. Work with a partner on the exercises below. Determine if the area formula  $A = \frac{1}{2}bh$  is always correct. You may use a calculator, but be sure to record your work on your paper as well. Figures are not drawn to scale.

	Area of Two Right Triangles	Area of Entire Triangle
		
		
		
		

2. Can we use the formula  $A = \frac{1}{2} \times \text{base} \times \text{height}$  to calculate the area of triangles that are not right triangles? Explain your thinking.

3. Examine the given triangle and expression.

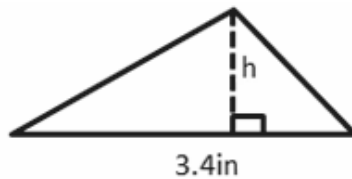


$$\frac{1}{2} (11 \text{ ft.})(4 \text{ ft.})$$

Explain what each part of the expression represents according to the triangle.

4. Joe found the area of a triangle by writing  $A = \frac{1}{2} (11 \text{ in.})(4 \text{ in.})$ , while Kaitlyn found the area by writing  $A = \frac{1}{2} (3 \text{ in.})(4 \text{ in.}) + \frac{1}{2} (8 \text{ in.})(4 \text{ in.})$ . Explain how each student approached the problem.

5. The triangle below has an area of 4.76 sq. in. If the base is 3.4 in., let  $h$  be the height in inches.



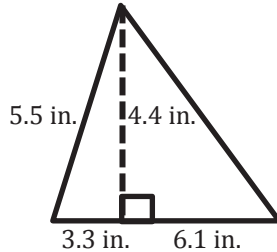
- a. Explain how the equation  $4.76 \text{ in}^2 = \frac{1}{2} (3.4 \text{ in.})h$  represents the situation.

- b. Solve the equation.

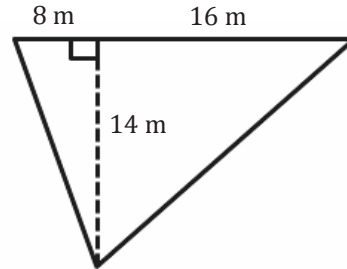
## Problem Set

Calculate the area of each shape below. Figures are not drawn to scale.

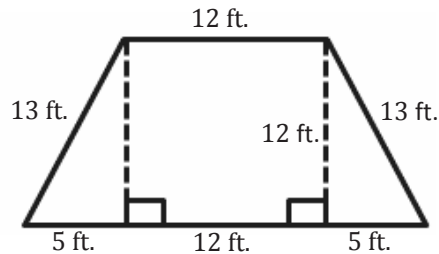
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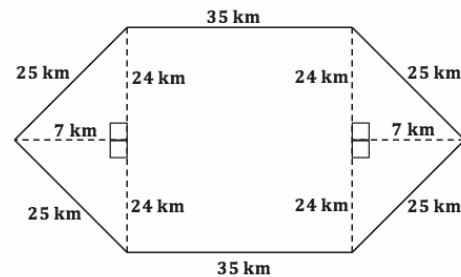
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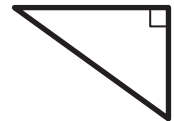
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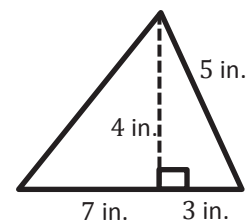
5. Immanuel is building a fence to make an enclosed play area for his dog. The enclosed area will be in the shape of a triangle with a base of 48 m. and an altitude of 32 m. How much space does the dog have to play?
6. Chauncey is building a storage bench for his son's playroom. The storage bench will fit into the corner and against two walls to form a triangle. Chauncey wants to buy a triangular shaped cover for the bench.

If the storage bench is  $2\frac{1}{2}$  ft. along one wall and  $4\frac{1}{4}$  ft. along the other wall, how big will the cover have to be to cover the entire bench?

Note: Figure is not to scale.



7. Examine the triangle to the right.
- Write an expression to show how you would calculate the area.
  - Identify each part of your expression as it relates to the triangle.



8. The floor of a triangular room has an area of  $32\frac{1}{2}$  sq. m. If the triangle's altitude is  $7\frac{1}{2}$  m, write an equation to determine the length of the base,  $b$ , in meters. Then solve the equation.

## Lesson 4: The Area of All Triangles Using Height and Base

### Classwork

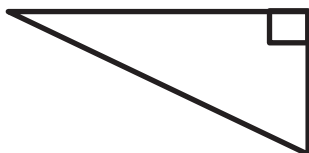
#### Opening Exercise

Draw and label the altitude of each triangle below.

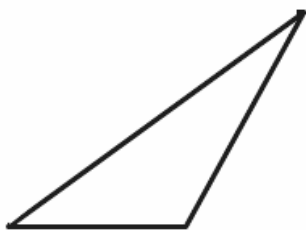
a.



b.



c.



#### Exploratory Challenge/Exercises 1–5

1. Use rectangle X and the triangle with the altitude inside (triangle X) to show that the area formula for the triangle is  $A = \frac{1}{2} \times \text{base} \times \text{height}$ .
  - a. Step One: Find the area of rectangle X.
  - b. Step Two: What is half the area of rectangle X?

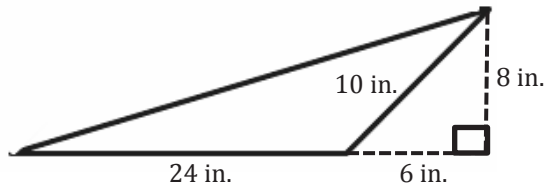
- c. Step Three: Prove, by decomposing triangle X, that it is the same as half of rectangle X. Please glue your decomposed triangle onto a separate sheet of paper. Glue it into rectangle X. What conclusions can you make about the triangle's area compared to the rectangle's area?
2. Use rectangle Y and the triangle with a side that is the altitude (triangle Y) to show the area formula for the triangle is  $A = \frac{1}{2} \times \text{base} \times \text{height}$ .
- a. Step One: Find the area of rectangle Y.
- b. Step Two: What is half the area of rectangle Y?
- c. Step Three: Prove, by decomposing triangle Y, that it is the same as half of rectangle Y. Please glue your decomposed triangle onto a separate sheet of paper. Glue it into rectangle Y. What conclusions can you make about the triangle's area compared to the rectangle's area?
3. Use rectangle Z and the triangle with the altitude outside (triangle Z) to show the area formula for the triangle is  $A = \frac{1}{2} \times \text{base} \times \text{height}$ .
- a. Step One: Find the area of rectangle Z.
- b. Step Two: What is half the area of rectangle Z?
- c. Step Three: Prove, by decomposing triangle Z, that it is the same as half of rectangle Z. Please glue your decomposed triangle onto a separate sheet of paper. Glue it into rectangle Z. What conclusions can you make about the triangle's area compared to the rectangle's area?

4. When finding the area of a triangle, does it matter where the altitude is located?
5. How can you determine which part of the triangle is the base and which is the height?

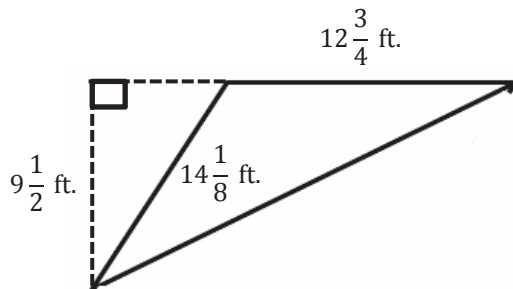
**Exercises 6–8**

Calculate the area of each triangle. Figures are not drawn to scale.

6.



7.



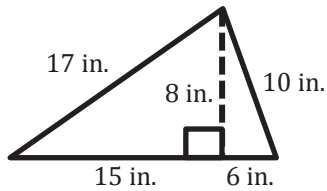
8. Draw three triangles (acute, right, and obtuse) that have the same area. Explain how you know they have the same area.



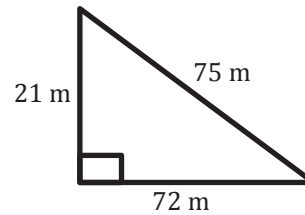
## Problem Set

Calculate the area of each figure below. Figures are not drawn to scale.

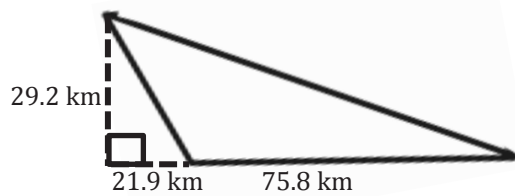
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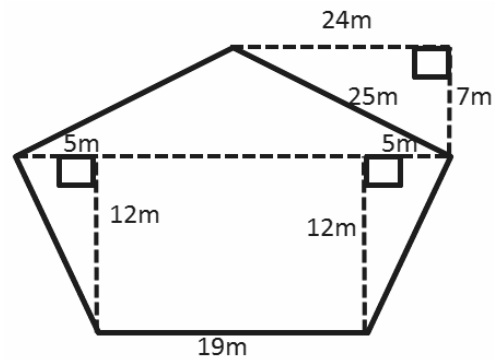
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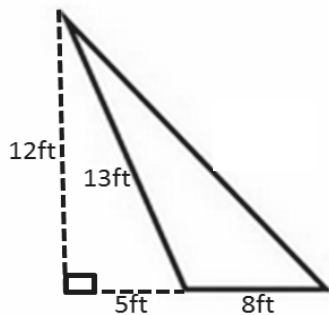


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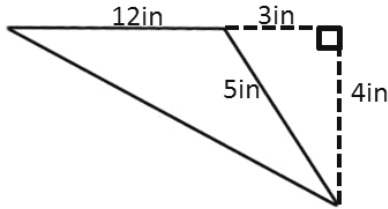


5. The Andersons are going on a long sailing trip during the summer. However, one of the sails on their sailboat ripped, and they have to replace it. The sail is pictured below.

If the sailboat sails are on sale for \$2 per square foot, how much will the new sail cost?



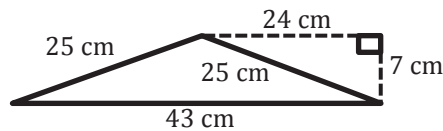
6. Darnell and Donovan are both trying to calculate the area of an obtuse triangle. Examine their calculations below.



Darnell's Work	Donovan's Work
$A = \frac{1}{2} \times 3 \text{ in.} \times 4 \text{ in.}$ $A = 6 \text{ in}^2$	$A = \frac{1}{2} \times 12 \text{ in.} \times 4 \text{ in.}$ $A = 24 \text{ in}^2$

Which student calculated the area correctly? Explain why the other student is not correct.

7. Russell calculated the area of the triangle below. His work is shown.

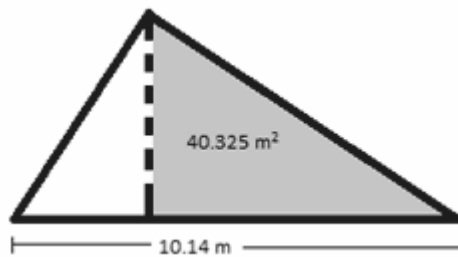


$$A = \frac{1}{2} \times 43 \text{ cm} \times 7 \text{ cm}$$

$$A = 150.5 \text{ cm}^2$$

Although Russell was told his work is correct, he had a hard time explaining why it is correct. Help Russell explain why his calculations are correct.

8. The larger triangle below has a base of 10.14 m; the gray triangle has an area of  $40.325 \text{ m}^2$ .



- Determine the area of the larger triangle if it has a height of 12.2 m.
- Let  $A$  be the area of the unshaded (white) triangle in square meters. Write and solve an equation to determine the value of  $A$ , using the areas of the larger triangle and the gray triangle.

## Lesson 5: The Area of Polygons Through Composition and Decomposition

### Classwork

#### Opening Exercise

Here is an aerial view of a woodlot.

If  $AB = 10$  units,  $FE = 8$  units,  $AF = 6$  units, and  $DE = 7$  units, find the lengths of the other two sides.

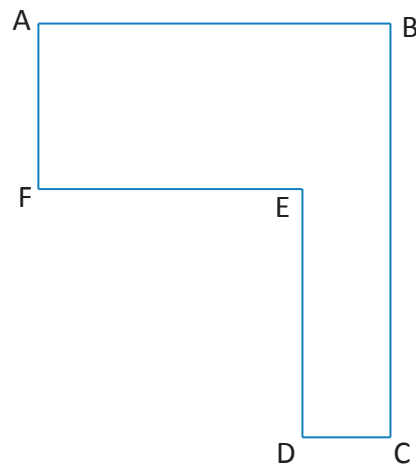
$DC =$

$BC =$

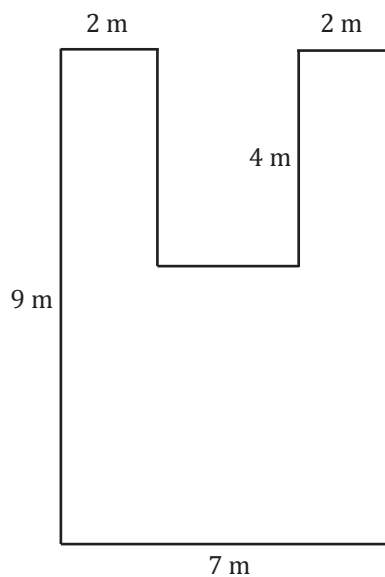
If  $DC = 10$  units,  $FE = 30$  units,  $AF = 28$  units, and  $BC = 54$  units, find the lengths of the other two sides.

$AB =$

$DE =$



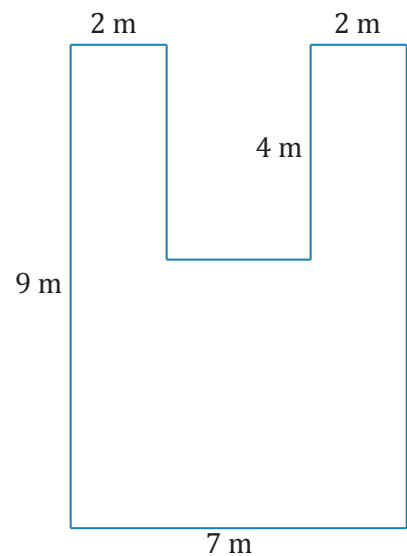
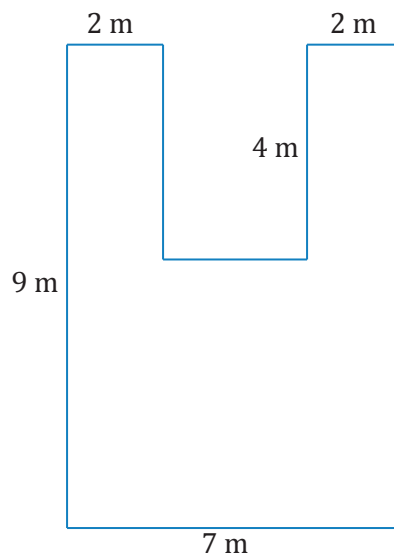
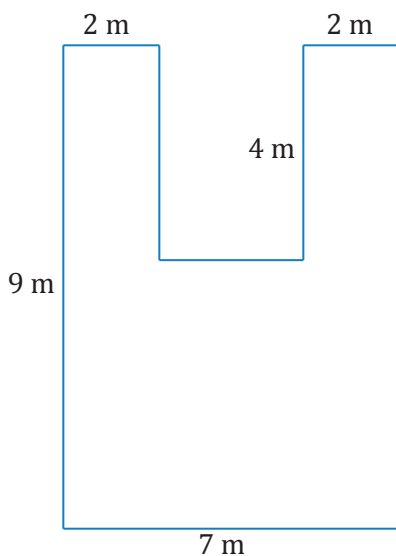
#### Discussion



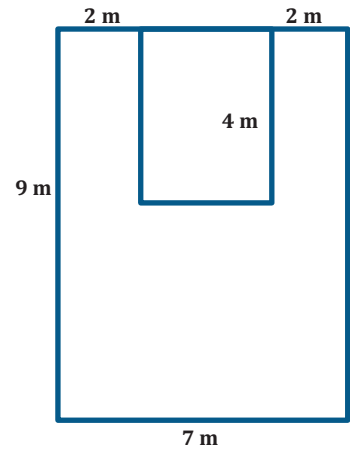
**Example 1: Decomposing Polygons into Rectangles**

The Intermediate School is producing a play that needs a special stage built. A diagram of the stage is shown below (not to scale).

- On the first diagram, divide the stage into three rectangles using two horizontal lines. Find the dimensions of these rectangles, and calculate the area of each. Then, find the total area of the stage.
- On the second diagram, divide the stage into three rectangles using two vertical lines. Find the dimensions of these rectangles, and calculate the area of each. Then, find the total area of the stage.
- On the third diagram, divide the stage into three rectangles using one horizontal line and one vertical line. Find the dimensions of these rectangles, and calculate the area of each. Then, find the total area of the stage.



- d. Think of this as a large rectangle with a piece removed.
- What are the dimensions of the large rectangle and the small rectangle?
  - What are the areas of the two rectangles?
  - What operation is needed to find the area of the original figure?
  - What is the difference in area between the two rectangles?
  - What do you notice about your answers to (a), (b), (c), and (d)?
  - Why do you think this is true?

**Example 2: Decomposing Polygons into Rectangles and Triangles**

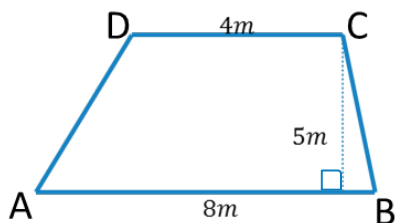
Parallelogram  $ABCD$  is part of a large solar power collector. The base measures 6 m and the height is 4 m.



- Draw a diagonal from  $A$  to  $C$ . Find the area of both triangles  $ABC$  and  $ACD$ .
- Draw in the other diagonal, from  $B$  to  $D$ . Find the area of both triangles  $ABD$  and  $BCD$ .

**Example 3: Decomposing Trapezoids**

The trapezoid below is a scale drawing of a garden plot.



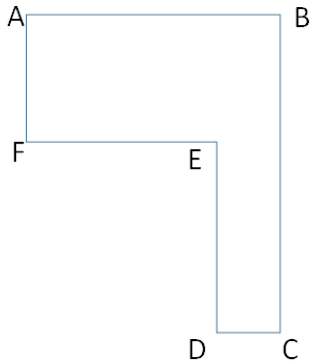
Find the area of both triangles  $ABC$  and  $ACD$ . Then find the area of the trapezoid.

Find the area of both triangles  $ABD$  and  $BCD$ . Then find the area of the trapezoid.

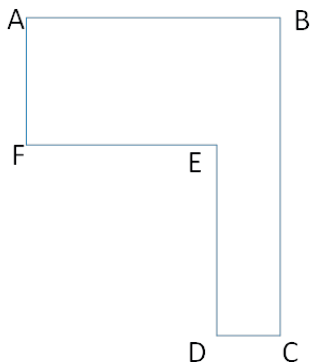
How else could we find this area?

## Problem Set

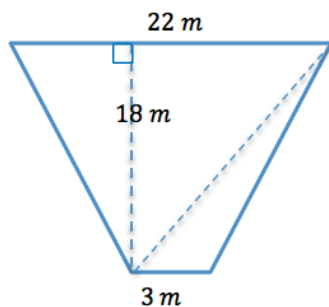
1. If  $AB = 20$  units,  $FE = 12$  units,  $AF = 9$  units, and  $DE = 12$  units, find the length of the other two sides. Then, find the area of the irregular polygon.



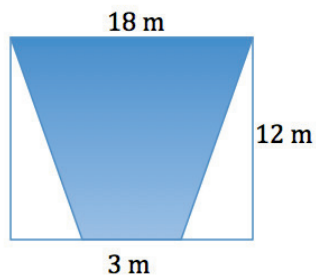
2. If  $DC = 1.9$  cm,  $FE = 5.6$  cm,  $AF = 4.8$  cm, and  $BC = 10.9$  cm, find the length of the other two sides. Then, find the area of the irregular polygon.



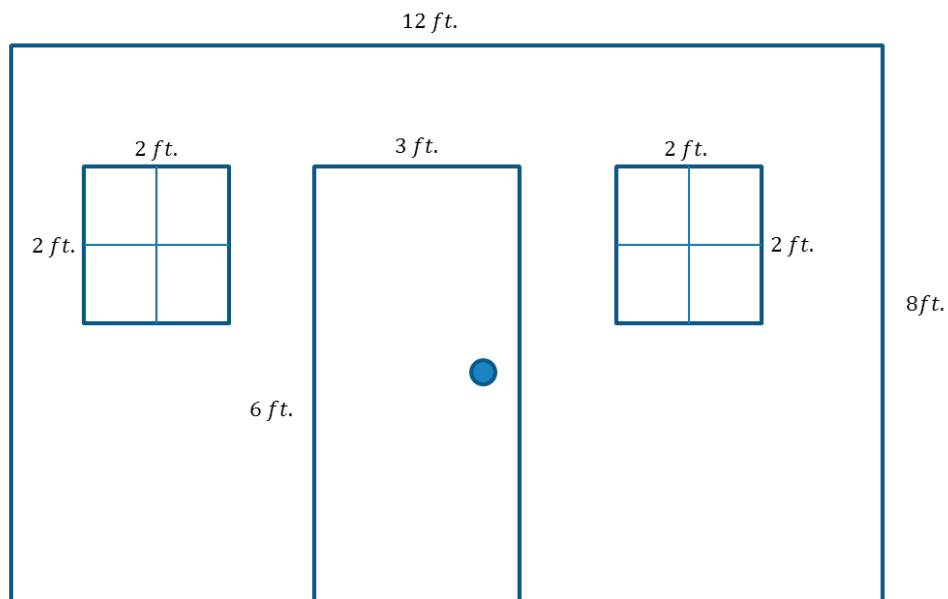
3. Determine the area of the trapezoid below. The trapezoid is not drawn to scale.



4. Determine the area of the shaded isosceles trapezoid below. The image is not drawn to scale.



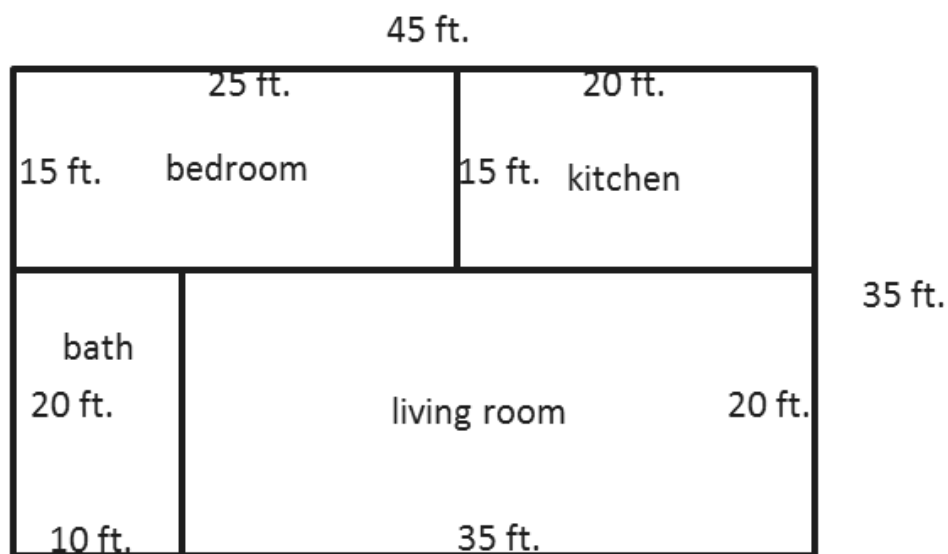
5. Here is a sketch of a wall that needs to be painted:



- The windows and door will not be painted. Calculate the area of the wall that will be painted.
- If a quart of Extra-Thick Goopy Sparkle paint covers  $30 \text{ ft}^2$ , how many quarts must be purchased for the painting job?



6. The figure below shows a floor plan of a new apartment. New carpeting has been ordered, which will cover the living room and bedroom but not the kitchen or bathroom. Determine the carpeted area by composing or decomposing in two different ways, and then explain why they are equivalent.



## Lesson 6: Area in the Real World

### Classwork

#### Exploratory Challenge 1: Classroom Wall Paint

The custodians are considering painting our classroom next summer. In order to know how much paint they must buy, the custodians need to know the total surface area of the walls. Why do you think they need to know this, and how can we find the information?

Make a prediction of how many square feet of painted surface there are on one wall in the room. If the floor has square tiles, these can be used as a guide.

Estimate the dimensions and the area. Predict the area before you measure.

My prediction: \_\_\_\_\_  $\text{ft}^2$ .

- Measure and sketch one classroom wall. Include measurements of windows, doors, or anything else that would not be painted.

*Sketch:*

Object or Item to Be Measured	Measurement Units	Precision (measure to the nearest)	Length	Width	Expression that Shows the Area	Area
door	feet	half foot	$6\frac{1}{2}$ ft.	$3\frac{1}{2}$ ft.	$6\frac{1}{2}$ ft. $\times$ $3\frac{1}{2}$ ft.	$22\frac{3}{4}$ ft <sup>2</sup>

- b. Work with your partners and your sketch of the wall to determine the area that needs paint. Show your sketch and calculations below; clearly mark your measurements and area calculations.

- c. A gallon of paint covers about 350 ft<sup>2</sup>. Write an expression that shows the total area of the wall. Evaluate it to find how much paint is needed to paint the wall.



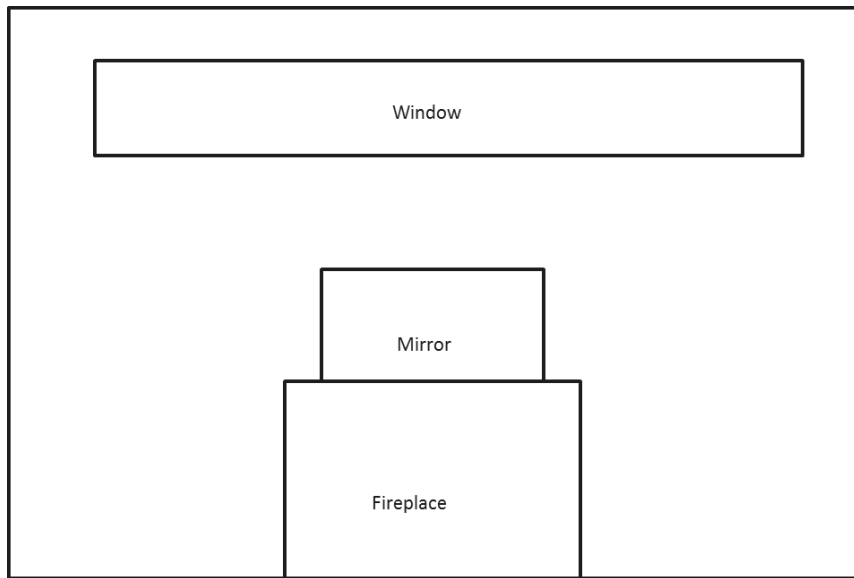
- d. How many gallons of paint would need to be purchased to paint the wall?

Exploratory Challenge 2

Object or Item to Be Measured	Measurement Units	Precision (measure to the nearest)	Length	Width	Area
door	feet	half foot	$6\frac{1}{2}$ ft.	$3\frac{1}{2}$ ft.	$22\frac{3}{4}$ ft <sup>2</sup>

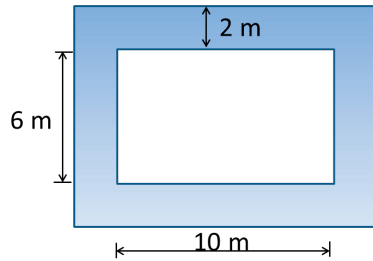
## Problem Set

1. Below is a drawing of a wall that is to be covered with either wallpaper or paint. The wall is 8 ft. high and 16 ft. wide. The window, mirror, and fireplace are not to be painted or papered. The window measures 18 in. wide and 14 ft. high. The fireplace is 5 ft. wide and 3 ft. high, while the mirror above the fireplace is 4 ft. wide and 2 ft. high. (Note: this drawing is not to scale.)

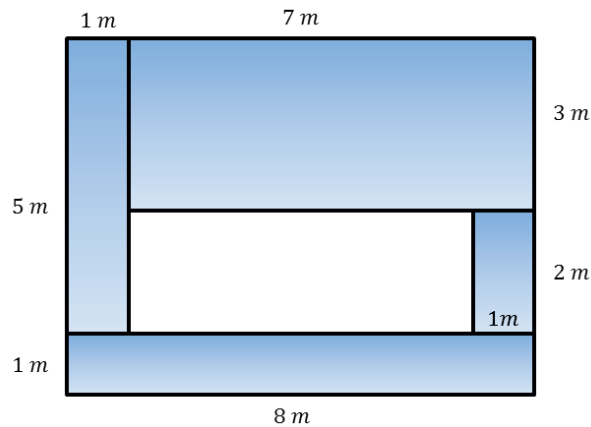


- How many square feet of wallpaper are needed to cover the wall?
  - The wallpaper is sold in rolls that are 18 in. wide and 33 ft. long. Rolls of solid color wallpaper will be used, so patterns do not have to match up.
    - What is the area of one roll of wallpaper?
    - How many rolls would be needed to cover the wall?
  - This week, the rolls of wallpaper are on sale for \$11.99/roll. Find the cost of covering the wall with wallpaper.
  - A gallon of special textured paint covers  $200 \text{ ft}^2$  and is on sale for \$22.99/gallon. The wall needs to be painted twice (the wall needs two coats of paint). Find the cost of using paint to cover the wall.
2. A classroom has a length of 30 ft. and a width of 20 ft. The flooring is to be replaced by tiles. If each tile has a length of 36 in. and a width of 24 in., how many tiles are needed to cover the classroom floor?
3. Challenge: Assume that the tiles from Problem 2 are unavailable. Another design is available, but the tiles are square, 18 in. on a side. If these are to be installed, how many must be ordered?

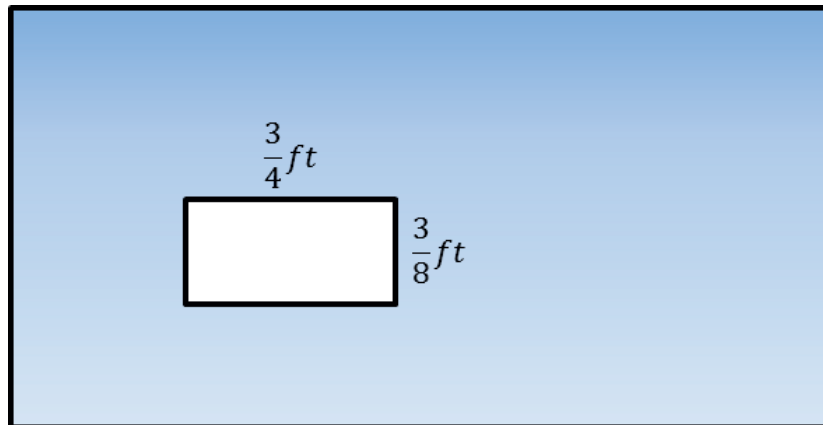
4. A rectangular flower bed measures 10 m by 6 m. It has a path 2 m wide around it. Find the area of the path.



5. A diagram of Tracy's deck is shown below, shaded blue. He wants to cover the missing portion of his deck with soil in order to grow a garden.
- a. Find the area of the missing portion of the deck. Write the expression and evaluate it.



- b. Find the missing portion of the deck using a different method. Write the expression and evaluate it.
- c. Write two equivalent expressions that can be used to determine the area of the missing portion of the deck.
- d. Explain how each expression demonstrates a different understanding of the diagram.
6. The entire large rectangle below has an area of  $3\frac{1}{2} \text{ ft}^2$ . If the dimensions of the white rectangle are as shown below, write and solve an equation to find the area,  $A$ , of the shaded region.



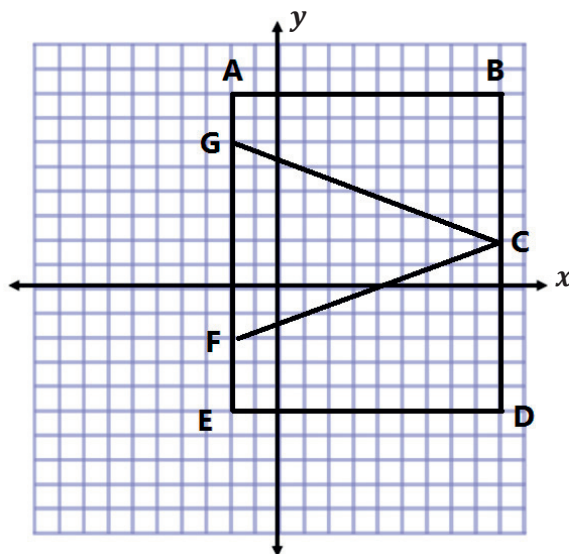
## Lesson 7: Distance on the Coordinate Plane

### Classwork

#### Example

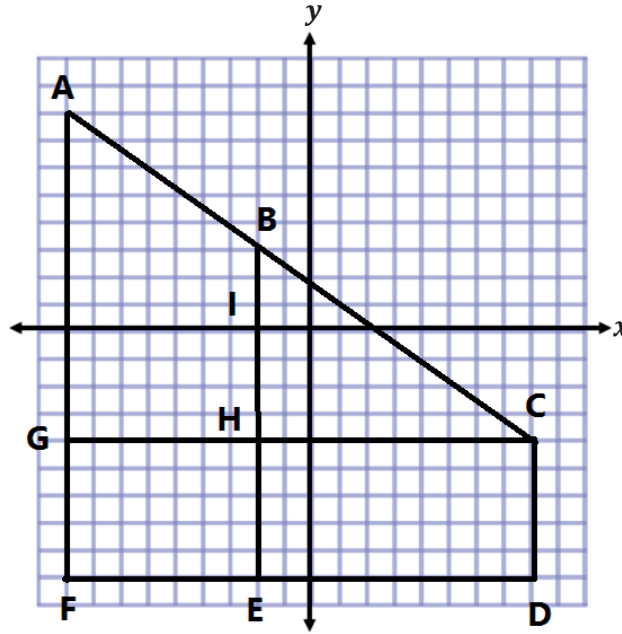
Determine the lengths of the given line segments by determining the distance between the two endpoints.

Line Segment	Point	Point	Distance	Proof
$\overline{AB}$				
$\overline{BC}$				
$\overline{CD}$				
$\overline{BD}$				
$\overline{DE}$				
$\overline{EF}$				
$\overline{FG}$				
$\overline{EG}$				
$\overline{GA}$				
$\overline{FA}$				
$\overline{EA}$				



## Exercise

Complete the table using the diagram on the coordinate plane.



Line Segment	Point	Point	Distance	Proof
$\overline{BI}$				
$\overline{BH}$				
$\overline{BE}$				
$\overline{GH}$				
$\overline{HC}$				
$\overline{GC}$				
$\overline{CD}$				
$\overline{FG}$				
$\overline{GA}$				
$\overline{AF}$				



**Extension**

For each problem below, write the coordinates of two points that are 5 units apart with the segment connecting these points having the following characteristics.

- a. The segment is vertical.
  
  
  
  
  
  
  
  
  
  
- b. The segment intersects the  $x$ -axis.
  
  
  
  
  
  
  
  
  
  
- c. The segment intersects the  $y$ -axis.
  
  
  
  
  
  
  
  
  
  
- d. The segment is vertical and lies above the  $x$ -axis.

## Problem Set

1. Given the pairs of points, determine whether the segment that joins them is horizontal, vertical, or neither.

a.  $X(3, 5)$  and  $Y(-2, 5)$  \_\_\_\_\_

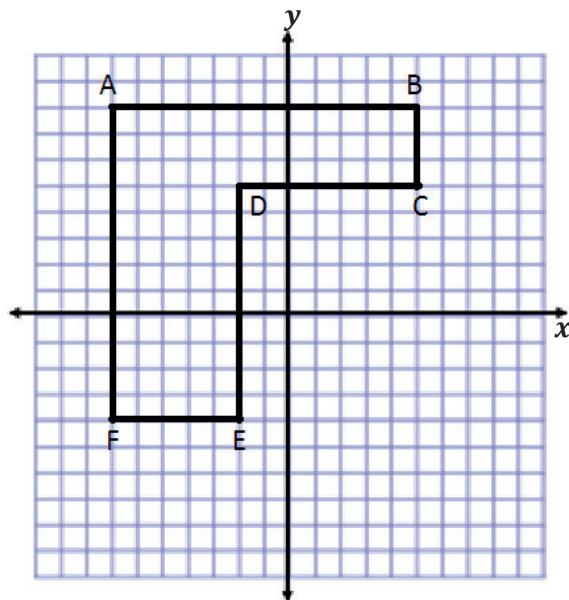
b.  $M(-4, 9)$  and  $N(4, -9)$  \_\_\_\_\_

c.  $E(-7, 1)$  and  $F(-7, 4)$  \_\_\_\_\_

2. Complete the table using absolute value to determine the lengths of the line segments.

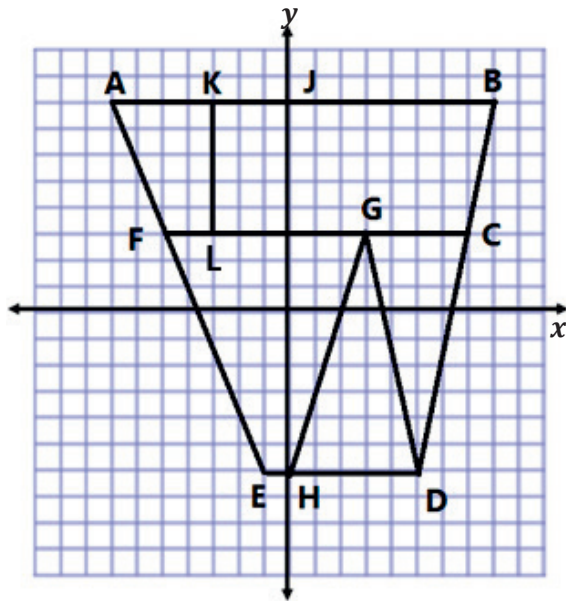
Line Segment	Point	Point	Distance	Proof
$\overline{AB}$	$(-3, 5)$	$(7, 5)$		
$\overline{CD}$	$(1, -3)$	$(-6, -3)$		
$\overline{EF}$	$(2, -9)$	$(2, -3)$		
$\overline{GH}$	$(6, 1)$	$(6, 16)$		
$\overline{JK}$	$(-3, 0)$	$(-3, 12)$		

3. Complete the table using the diagram and absolute value to determine the lengths of the line segments.



Line Segment	Point	Point	Distance	Proof
$\overline{AB}$				
$\overline{BC}$				
$\overline{CD}$				
$\overline{DE}$				
$\overline{EF}$				
$\overline{FA}$				

4. Complete the table using the diagram and absolute value to determine the lengths of the line segments.



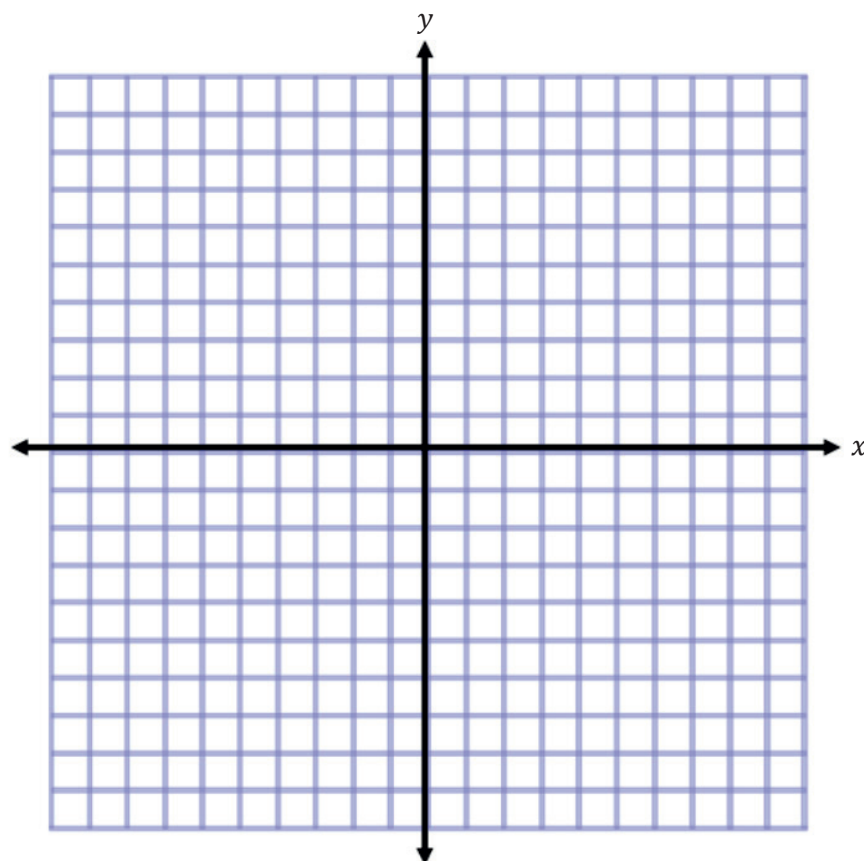
Line Segment	Point	Point	Distance	Proof
$\overline{AB}$				
$\overline{CG}$				
$\overline{CF}$				
$\overline{GF}$				
$\overline{DH}$				
$\overline{DE}$				
$\overline{HJ}$				
$\overline{KL}$				

5. Name two points in different quadrants that form a vertical line segment that is 8 units in length.
6. Name two points in the same quadrant that form a horizontal line segment that is 5 units in length.

## Lesson 8: Drawing Polygons in the Coordinate Plane

### Classwork

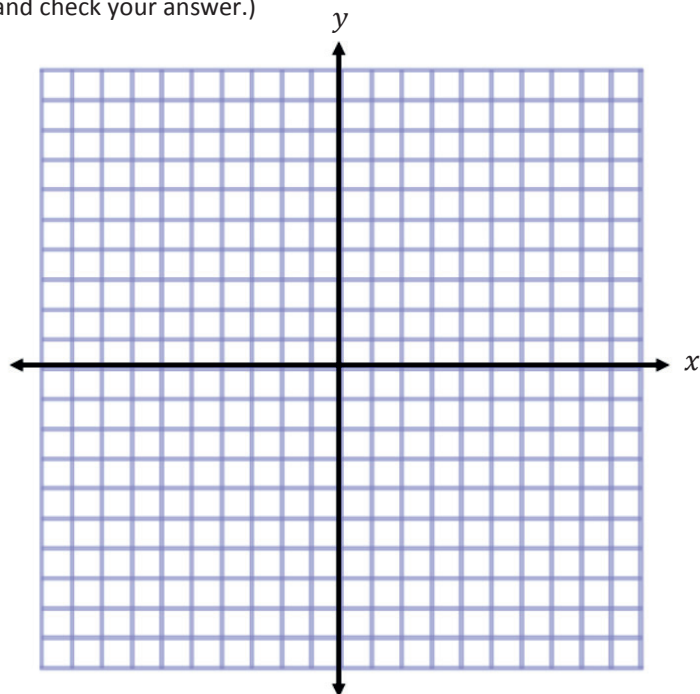
#### Examples



1. Plot and connect the points  $A(3, 2)$ ,  $B(3, 7)$ , and  $C(8, 2)$ . Name the shape, and determine the area of the polygon.

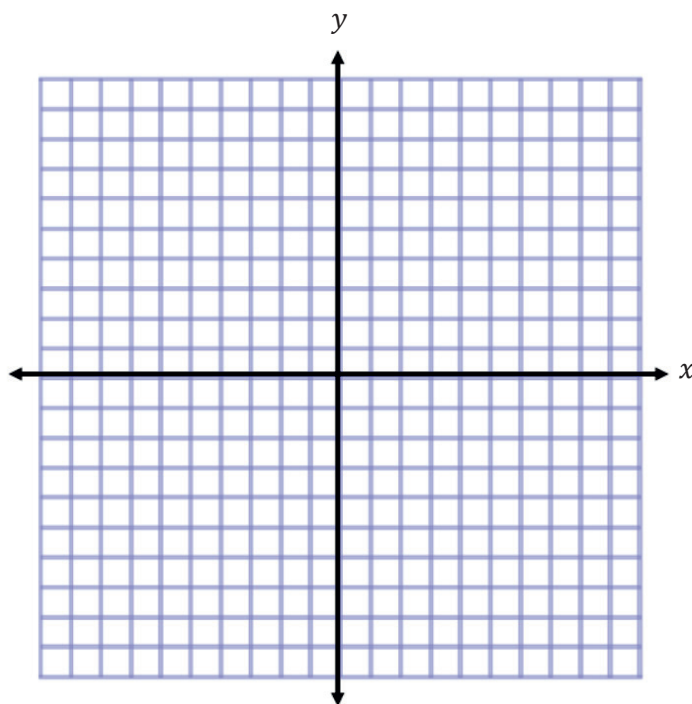
- Plot and connect the points  $E(-8, 8)$ ,  $F(-2, 5)$ , and  $G(-7, 2)$ . Then give the best name for the polygon, and determine the area.
- Plot and connect the following points:  $K(-9, -7)$ ,  $L(-4, -2)$ ,  $M(-1, -5)$ , and  $N(-5, -5)$ . Give the best name for the polygon, and determine the area.
- Plot and connect the following points:  $P(1, -4)$ ,  $Q(5, -2)$ ,  $R(9, -4)$ ,  $S(7, -8)$ , and  $T(3, -8)$ . Give the best name for the polygon, and determine the area.

5. Two of the coordinates of a rectangle are  $A(3, 7)$  and  $B(3, 2)$ . The rectangle has an area of 30 square units. Give the possible locations of the other two vertices by identifying their coordinates. (Use the coordinate plane to draw and check your answer.)



### Exercises

For Exercises 1 and 2, plot the points, name the shape, and determine the area of the shape. Then write an expression that could be used to determine the area of the figure. Explain how each part of the expression corresponds to the situation.



1.  $A(4, 6)$ ,  $B(8, 6)$ ,  $C(10, 2)$ ,  $D(8, -3)$ ,  $E(5, -3)$ , and  $F(2, 2)$

2.  $X(-9, 6)$ ,  $Y(-2, -1)$ , and  $Z(-8, -7)$

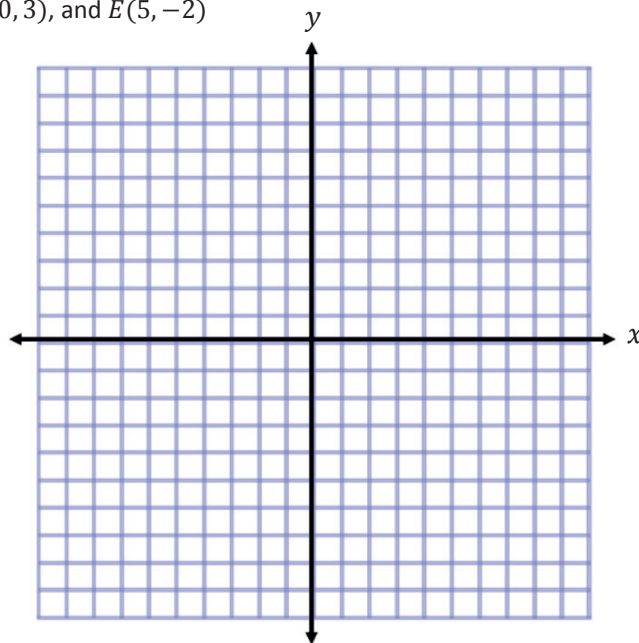
3. A rectangle with vertices located at  $(-3, 4)$  and  $(5, 4)$  has an area of 32 square units. Determine the location of the other two vertices.
4. Challenge: A triangle with vertices located at  $(-2, -3)$  and  $(3, -3)$  has an area of 20 square units. Determine one possible location of the other vertex.



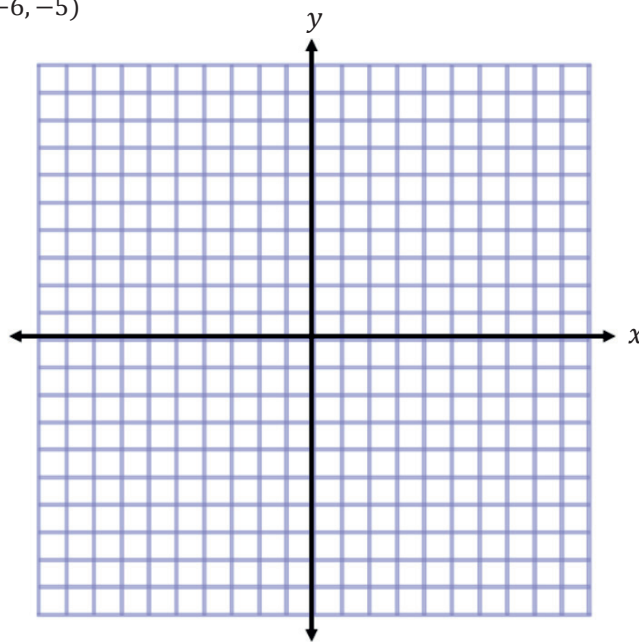
**Problem Set**

Plot the points for each shape, determine the area of the polygon, and then write an expression that could be used to determine the area of the figure. Explain how each part of the expression corresponds to the situation.

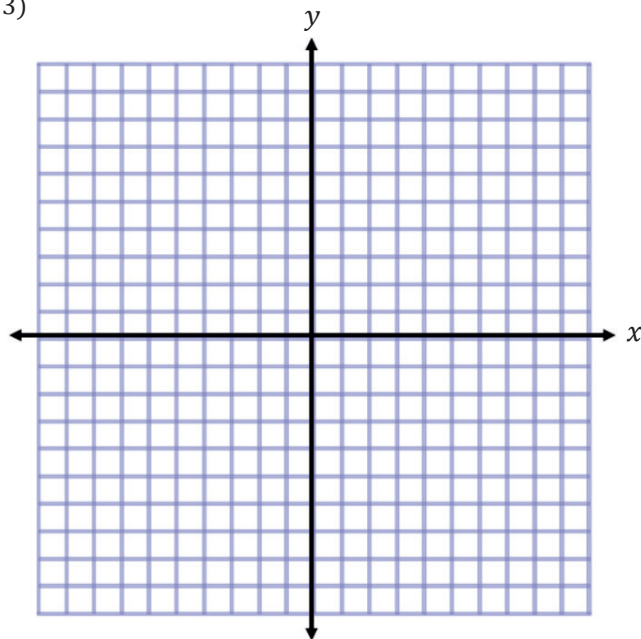
1.  $A(1, 3)$ ,  $B(2, 8)$ ,  $C(8, 8)$ ,  $D(10, 3)$ , and  $E(5, -2)$



2.  $X(-10, 2)$ ,  $Y(-3, 6)$ , and  $Z(-6, -5)$



3.  $E(5, 7)$ ,  $F(9, -5)$ , and  $G(1, -3)$



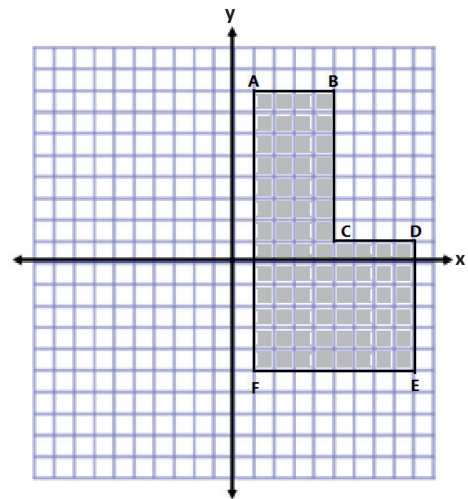
4. Find the area of the triangle in Problem 3 using a different method. Then, compare the expressions that can be used for both solutions in Problems 3 and 4.
5. Two vertices of a rectangle are  $(8, -5)$  and  $(8, 7)$ . If the area of the rectangle is 72 square units, name the possible location of the other two vertices.
6. A triangle with two vertices located at  $(5, -8)$  and  $(5, 4)$  has an area of 48 square units. Determine one possible location of the other vertex.

## Lesson 9: Determining Perimeter and Area of Polygons on the Coordinate Plane

### Classwork

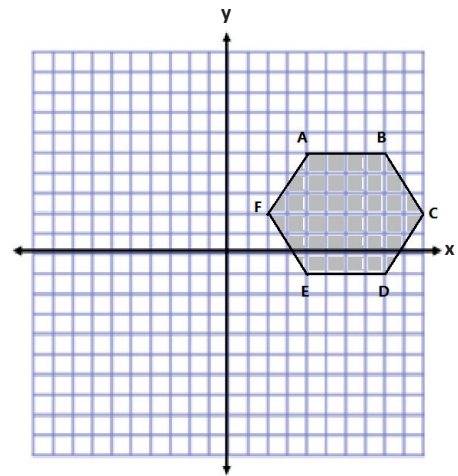
#### Example 1

Jasjeet has made a scale drawing of a vegetable garden she plans to make in her backyard. She needs to determine the perimeter and area to know how much fencing and dirt to purchase. Determine both the perimeter and area.



#### Example 2

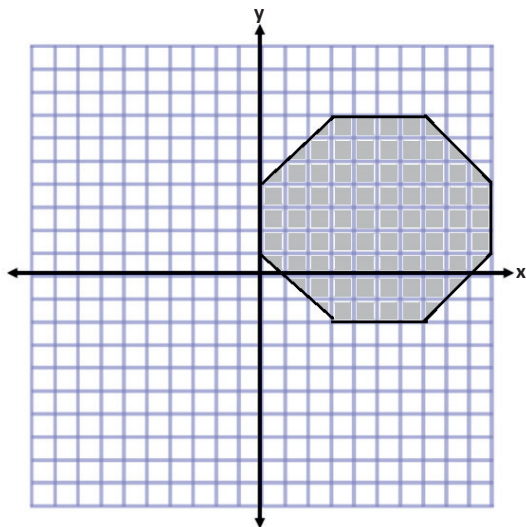
Calculate the area of the polygon using two different methods. Write two expressions to represent the two methods, and compare the structure of the expressions.



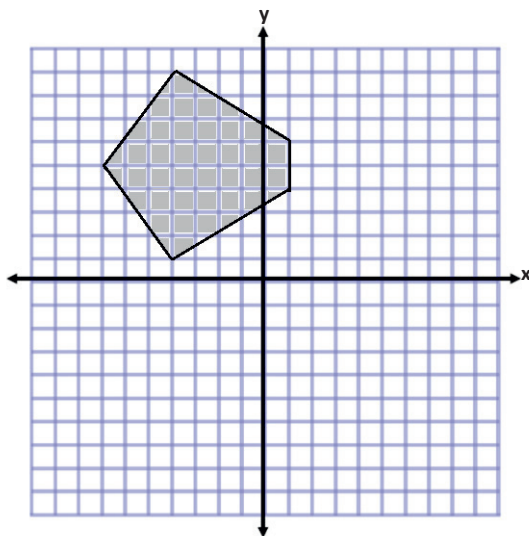
**Exercises**

1. Determine the area of the following shapes.

a.

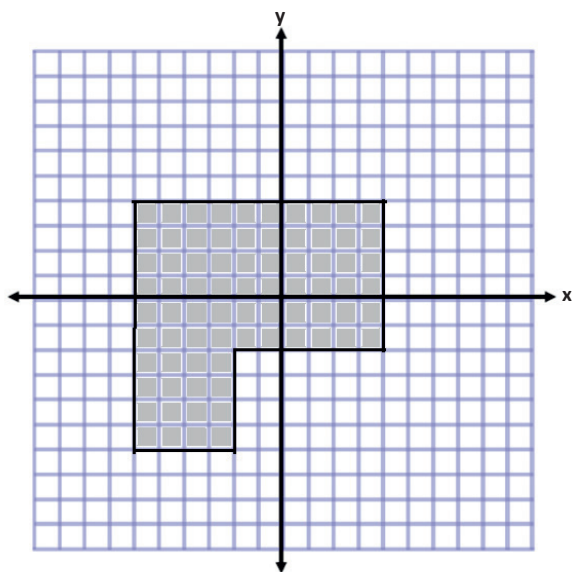


b.

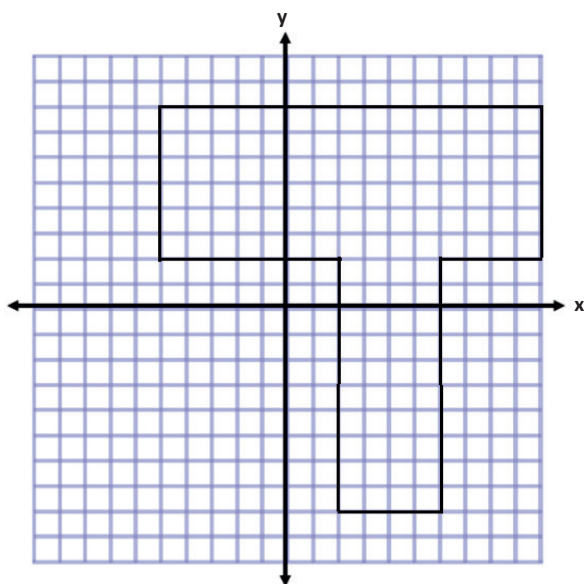


2. Determine the area and perimeter of the following shapes.

a.

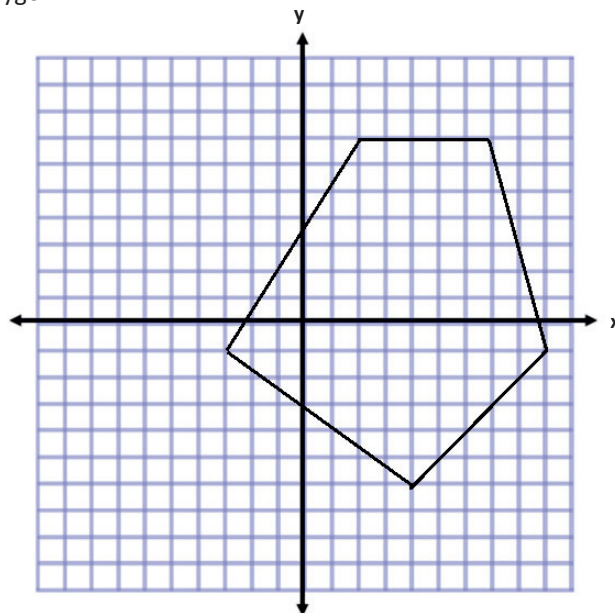


b.

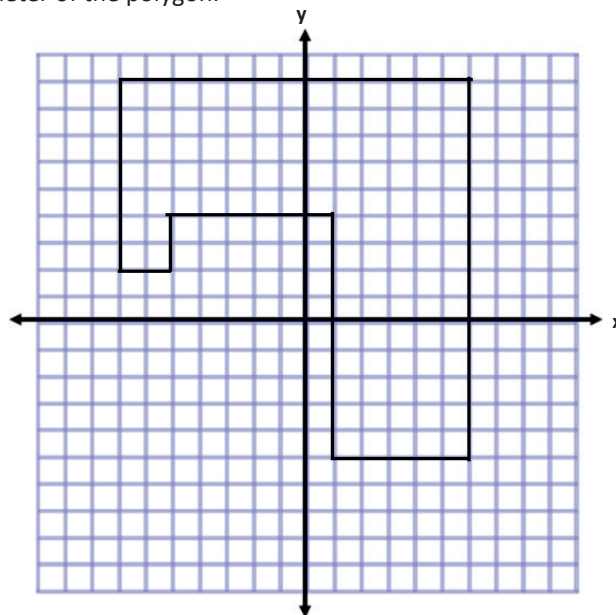


**Problem Set**

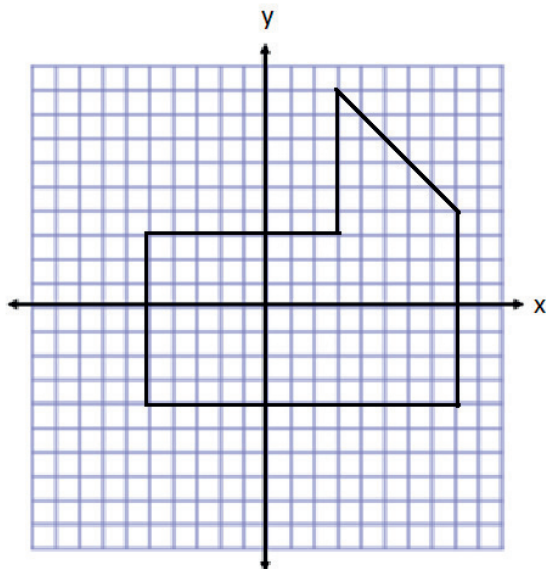
1. Determine the area of the polygon.



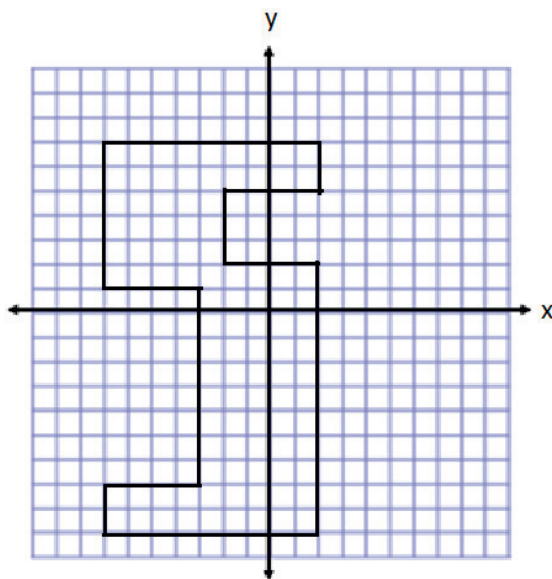
2. Determine the area and perimeter of the polygon.



3. Determine the area of the polygon. Then, write an expression that could be used to determine the area.

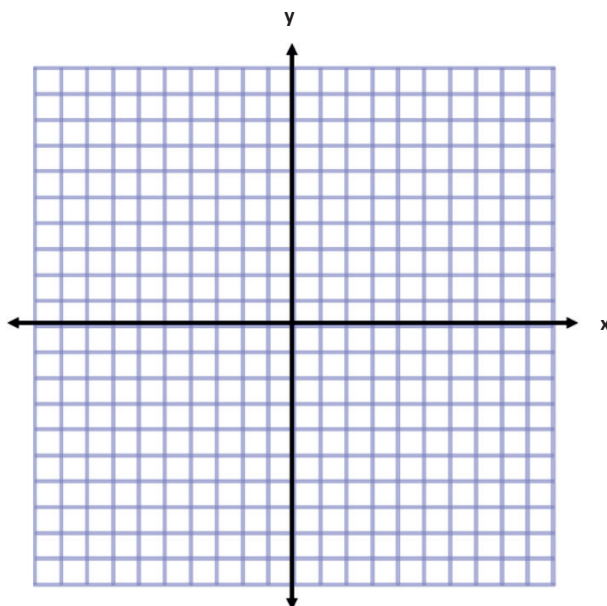


4. If the length of each square was worth 2 instead of 1, how would the area in Problem 3 change? How would your expression change to represent this area?
5. Determine the area of the polygon. Then, write an expression that represents the area.



6. Describe another method you could use to find the area of the polygon in Problem 5. Then, state how the expression for the area would be different than the expression you wrote.

7. Write one of the letters from your name using rectangles on the coordinate plane. Then, determine the area and perimeter. (For help see Exercise 2(b). This irregular polygon looks sort of like a T.)



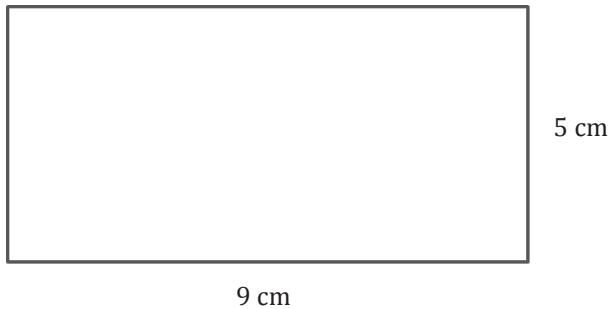


## Lesson 10: Distance, Perimeter, and Area in the Real World

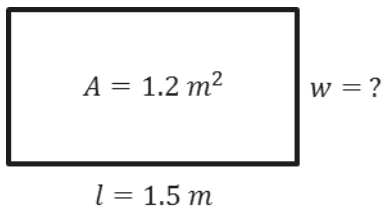
### Classwork

#### Opening Exercise

- a. Find the area and perimeter of this rectangle:



- b. Find the width of this rectangle. The area is  $1.2 \text{ m}^2$ , and the length is  $1.5 \text{ m}$ .



#### Example: Student Desks or Tables

1. Measure the dimensions of the top of your desk.
2. How do you find the area of the top of your desk?
3. How do you find the perimeter?
4. Record these on your paper in the appropriate column.

## Exploratory Challenge

Estimate and predict the area and perimeter of each object. Then measure each object, and calculate both the area and perimeter of each.

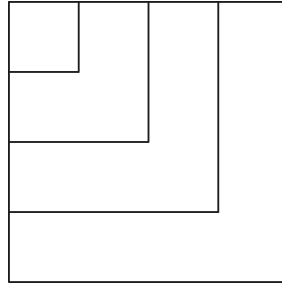
Object or Item to be Measured	Measurement Units	Precision (measure to the nearest)	Area Prediction (square units)	Area (square units) Write the expression and evaluate it.	Perimeter Prediction (linear units)	Perimeter (linear units)
Ex: door	feet	half foot		$6\frac{1}{2}\text{ ft.} \times 3\frac{1}{2}\text{ ft.}$ $= 22\frac{3}{4}\text{ ft}^2$		$2\left(3\frac{1}{2}\text{ ft.} + 6\frac{1}{2}\text{ ft.}\right)$ $= 20\text{ ft.}$
desktop						

## Optional Challenge

Object or Item to be Measured	Measurement Units	Precision (measure to the nearest)	Area (square units)	Perimeter (linear units)
Ex: door	feet	half foot	$6\frac{1}{2}\text{ ft.} \times 3\frac{1}{2}\text{ ft.}$ $= 22\frac{3}{4}\text{ ft}^2$	$2\left(3\frac{1}{2}\text{ ft.} + 6\frac{1}{2}\text{ ft.}\right)$ $= 20\text{ ft.}$

## Problem Set

1. How is the length of the side of a square related to its area and perimeter? The diagram below shows the first four squares stacked on top of each other with their upper left-hand corners lined up. The length of one side of the smallest square is 1 foot.

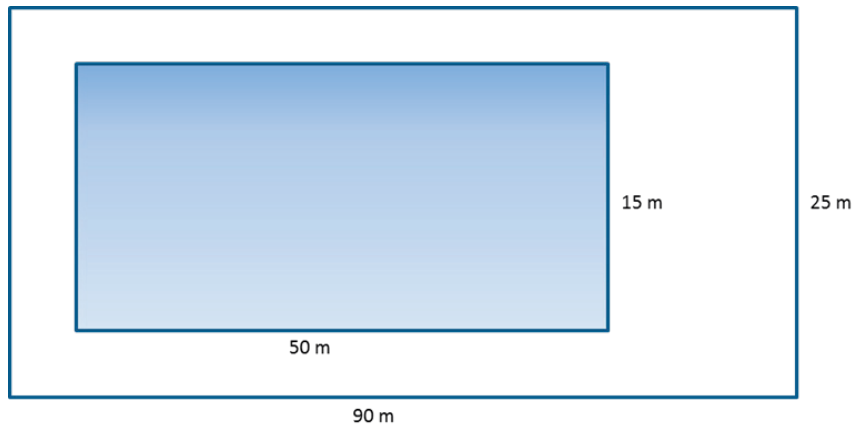


- a. Complete this chart calculating area and perimeter for each square.

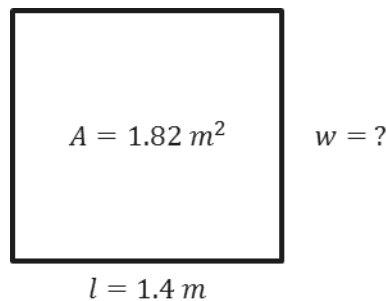
Side Length (in feet)	Expression Showing the Area	Area (in square feet)	Expression Showing the Perimeter	Perimeter (in feet)
1	$1 \times 1$	1	$1 \times 4$	4
2				
3				
4				
5				
6				
7				
8				
9				
10				
$n$				

- b. In a square, which numerical value is greater, the area or the perimeter?  
 c. When is the numerical value of a square's area (in square units) equal to its perimeter (in units)?  
 d. Why is this true?

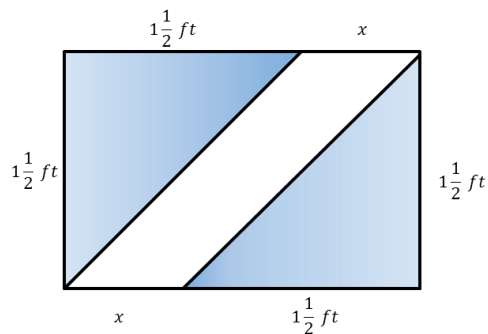
2. This drawing shows a school pool. The walkway around the pool needs special nonskid strips installed but only at the edge of the pool and the outer edges of the walkway.



- a. Find the length of nonskid strips that is needed for the job.
- b. The nonskid strips are sold only in rolls of 50 m. How many rolls need to be purchased for the job?
3. A homeowner called in a painter to paint the walls and ceiling of one bedroom. His bedroom is 18 ft. long, 12 ft. wide, and 8 ft. high. The room has two doors, each 3 ft. by 7 ft., and three windows each 3 ft. by 5 ft. The doors and windows will not be painted. A gallon of paint can cover  $300 \text{ ft}^2$ . A hired painter claims he needs a minimum of 4 gallons. Show that his estimate is too high.
4. Theresa won a gardening contest and was awarded a roll of deer-proof fencing. The fencing is 36 feet long. She and her husband, John, discuss how to best use the fencing to make a rectangular garden. They agree that they should only use whole numbers of feet for the length and width of the garden.
- a. What are all of the possible dimensions of the garden?
- b. Which plan yields the maximum area for the garden? Which plan yields the minimum area?
5. Write and then solve the equation to find the missing value below.



6. Challenge: This is a drawing of the flag of the Republic of the Congo. The area of this flag is  $3\frac{3}{4}\text{ ft}^2$ .



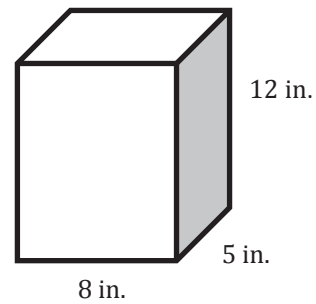
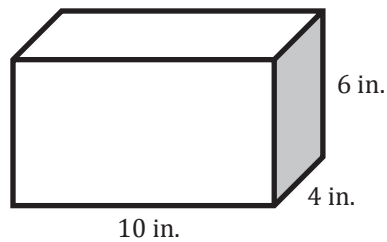
- Using the area formula, tell how you would determine the value of the base. This figure is not drawn to scale.
- Using what you found in part (a), determine the missing value of the base.

## Lesson 11: Volume with Fractional Edge Lengths and Unit Cubes

### Classwork

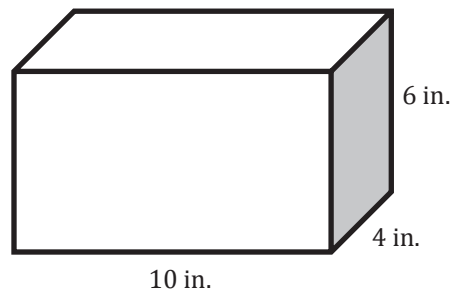
#### Opening Exercise

Which prism holds more  $1 \text{ in.} \times 1 \text{ in.} \times 1 \text{ in.}$  cubes? How many more cubes does the prism hold?



#### Example 1

A box with the same dimensions as the prism in the Opening Exercise is used to ship miniature dice whose side lengths have been cut in half. The dice are  $\frac{1}{2} \text{ in.} \times \frac{1}{2} \text{ in.} \times \frac{1}{2} \text{ in.}$  cubes. How many dice of this size can fit in the box?



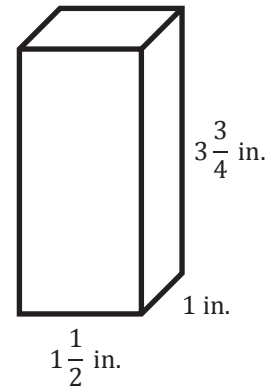
**Example 2**

A  $\frac{1}{4}$  in. cube was used to fill the prism.

How many  $\frac{1}{4}$  in. cubes does it take to fill the prism?

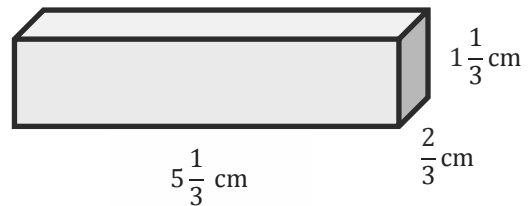
What is the volume of the prism?

How is the number of cubes related to the volume?

**Exercises**

1. Use the prism to answer the following questions.

a. Calculate the volume.



b. If you have to fill the prism with cubes whose side lengths are less than 1 cm, what size would be best?

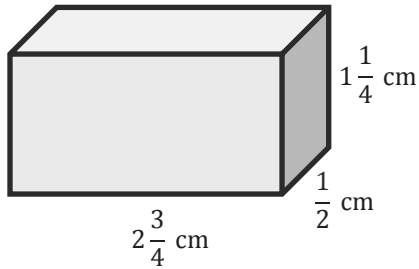
c. How many of the cubes would fit in the prism?

d. Use the relationship between the number of cubes and the volume to prove that your volume calculation is correct.

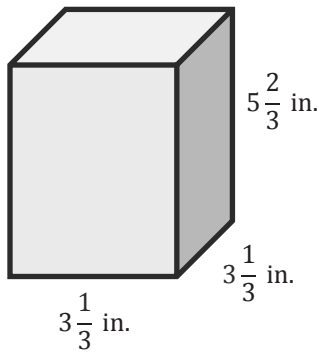


2. Calculate the volume of the following rectangular prisms.

a.



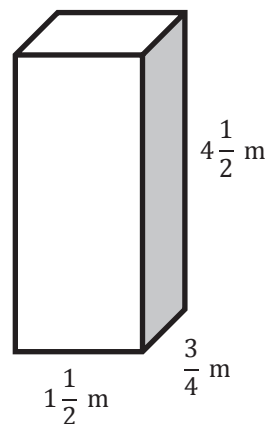
b.



3. A toy company is packaging its toys to be shipped. Each small toy is placed inside a cube-shaped box with side lengths of  $\frac{1}{2}$  in. These smaller boxes are then placed into a larger box with dimensions of  $12$  in.  $\times$   $4\frac{1}{2}$  in.  $\times$   $3\frac{1}{2}$  in.

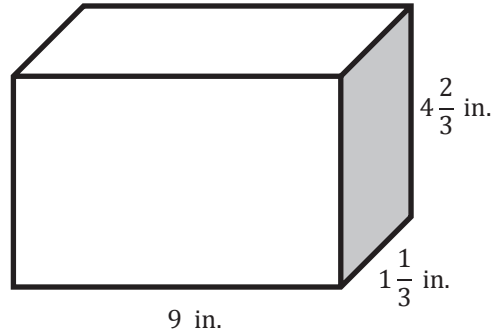
- a. What is the greatest number of small toy boxes that can be packed into the larger box for shipping?
- b. Use the number of small toy boxes that can be shipped in the larger box to help determine the volume of the shipping box.

4. A rectangular prism with a volume of 8 cubic units is filled with cubes twice: once with cubes with side lengths of  $\frac{1}{2}$  unit and once with cubes with side lengths of  $\frac{1}{3}$  unit.
- a. How many more of the cubes with  $\frac{1}{3}$ -unit side lengths than cubes with  $\frac{1}{2}$ -unit side lengths are needed to fill the prism?
- b. Why does it take more cubes with  $\frac{1}{3}$ -unit side lengths to fill the prism than it does with cubes with  $\frac{1}{2}$ -unit side lengths?
5. Calculate the volume of the rectangular prism. Show two different methods for determining the volume.



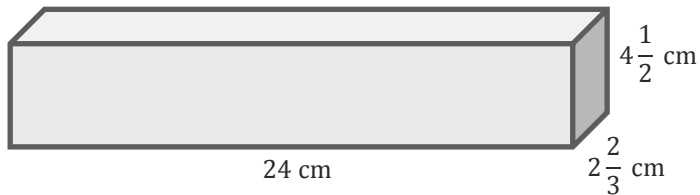
## Problem Set

1. Answer the following questions using this rectangular prism:

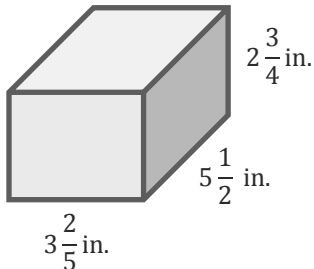


- What is the volume of the prism?
  - Linda fills the rectangular prism with cubes that have side lengths of  $\frac{1}{3}$  in. How many cubes does she need to fill the rectangular prism?
  - How is the number of cubes related to the volume?
  - Why is the number of cubes needed different from the volume?
  - Should Linda try to fill this rectangular prism with cubes that are  $\frac{1}{2}$  in. long on each side? Why or why not?
2. Calculate the volume of the following prisms.

a.



b.

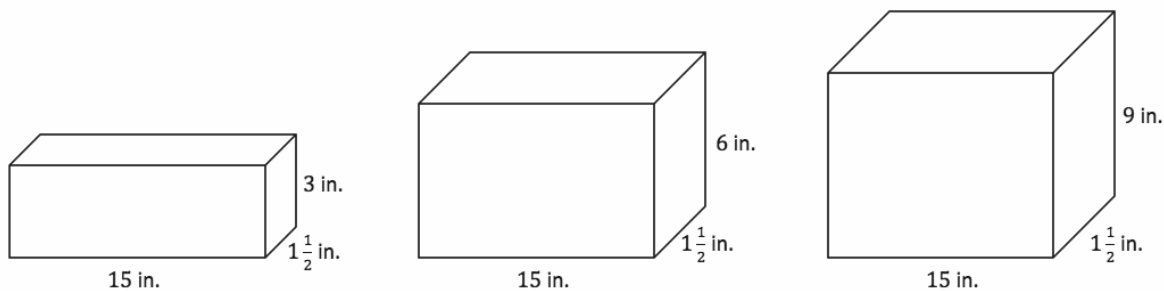


3. A rectangular prism with a volume of 12 cubic units is filled with cubes twice: once with cubes with  $\frac{1}{2}$ -unit side lengths and once with cubes with  $\frac{1}{3}$ -unit side lengths.
- How many more of the cubes with  $\frac{1}{3}$ -unit side lengths than cubes with  $\frac{1}{2}$ -unit side lengths are needed to fill the prism?
  - Finally, the prism is filled with cubes whose side lengths are  $\frac{1}{4}$  unit. How many  $\frac{1}{4}$ -unit cubes would it take to fill the prism?
4. A toy company is packaging its toys to be shipped. Each toy is placed inside a cube-shaped box with side lengths of  $3\frac{1}{2}$  in. These smaller boxes are then packed into a larger box with dimensions of 14 in.  $\times$  7 in.  $\times$   $3\frac{1}{2}$  in.
- What is the greatest number of toy boxes that can be packed into the larger box for shipping?
  - Use the number of toy boxes that can be shipped in the large box to determine the volume of the shipping box.
5. A rectangular prism has a volume of 34.224 cubic meters. The height of the box is 3.1 meters, and the length is 2.4 meters.
- Write an equation that relates the volume to the length, width, and height. Let  $w$  represent the width, in meters.
  - Solve the equation.

## Lesson 12: From Unit Cubes to the Formulas for Volume

### Classwork

#### Example 1



- Write a numerical expression for the volume of each of the rectangular prisms above.
- What do all of these expressions have in common? What do they represent?
- Rewrite the numerical expressions to show what they have in common.
- If we know volume for a rectangular prism as length times width times height, what is another formula for volume that we could use based on these examples?
- What is the area of the base for all of the rectangular prisms?

- f. Determine the volume of each rectangular prism using either method.
- g. How do the volumes of the first and second rectangular prisms compare? The volumes of the first and third?

**Example 2**

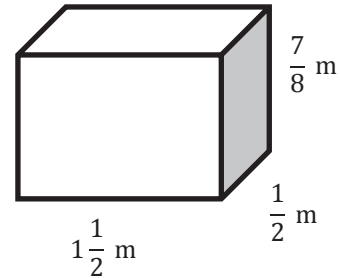
The base of a rectangular prism has an area of  $3\frac{1}{4}$  in<sup>2</sup>. The height of the prism is  $2\frac{1}{2}$  in. Determine the volume of the rectangular prism.

**Extension**

A company is creating a rectangular prism that must have a volume of 6 ft<sup>3</sup>. The company also knows that the area of the base must be  $2\frac{1}{2}$  ft<sup>2</sup>. How can you use what you learned today about volume to determine the height of the rectangular prism?

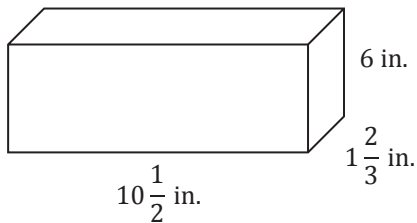
## Problem Set

1. Determine the volume of the rectangular prism.



2. The area of the base of a rectangular prism is  $4\frac{3}{4}\text{ft}^2$ , and the height is  $2\frac{1}{3}\text{ft}$ . Determine the volume of the rectangular prism.
3. The length of a rectangular prism is  $3\frac{1}{2}$  times as long as the width. The height is  $\frac{1}{4}$  of the width. The width is 3 cm. Determine the volume.

4.

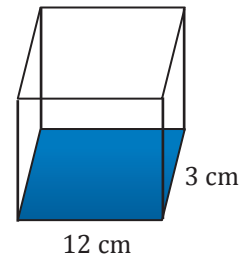


- a. Write numerical expressions to represent the volume in two different ways, and explain what each reveals.
- b. Determine the volume of the rectangular prism.
5. An aquarium in the shape of a rectangular prism has the following dimensions: length = 50 cm, width =  $25\frac{1}{2}$  cm, and height =  $30\frac{1}{2}$  cm.
- a. Write numerical expressions to represent the volume in two different ways, and explain what each reveals.
- b. Determine the volume of the rectangular prism.

6. The area of the base in this rectangular prism is fixed at  $36 \text{ cm}^2$ . As the height of the rectangular prism changes, the volume will also change as a result.

a. Complete the table of values to determine the various heights and volumes.

Height of Prism (in centimeters)	Volume of Prism (in cubic centimeters)
2	72
3	108
	144
	180
6	
7	
	288



- b. Write an equation to represent the relationship in the table. Be sure to define the variables used in the equation.
- c. What is the unit rate for this proportional relationship? What does it mean in this situation?
7. The volume of a rectangular prism is  $16.328 \text{ cm}^3$ . The height is  $3.14 \text{ cm}$ .
- a. Let  $B$  represent the area of the base of the rectangular prism. Write an equation that relates the volume, the area of the base, and the height.
- b. Solve the equation for  $B$ .



## Lesson 13: The Formulas for Volume

### Classwork

#### Example 1

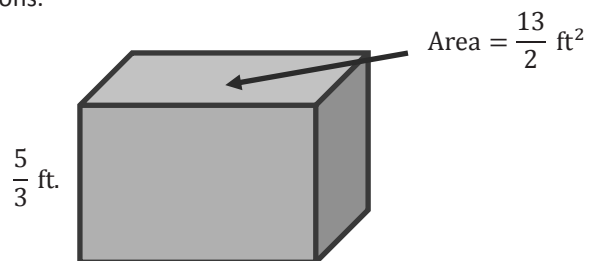
Determine the volume of a cube with side lengths of  $2\frac{1}{4}$  cm.

#### Example 2

Determine the volume of a rectangular prism with a base area of  $\frac{7}{12}$  ft<sup>2</sup> and a height of  $\frac{1}{3}$  ft.

### Exercises

1. Use the rectangular prism to answer the next set of questions.
  - a. Determine the volume of the prism.



- b. Determine the volume of the prism if the height of the prism is doubled.

- c. Compare the volume of the rectangular prism in part (a) with the volume of the prism in part (b). What do you notice?
- d. Complete and use the table below to determine the relationship between the height and volume.

Height of Prism (in feet)	Volume of Prism (in cubic feet)
$\frac{5}{3}$	$\frac{65}{6}$
$\frac{10}{3}$	$\frac{130}{6}$
$\frac{15}{3}$	
$\frac{20}{3}$	

What happened to the volume when the height was tripled?

What happened to the volume when the height was quadrupled?

What conclusions can you make when the base area stays constant and only the height changes?

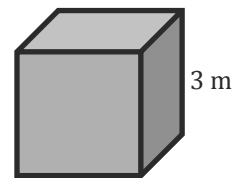
2.

- a. If  $B$  represents the area of the base and  $h$  represents the height, write an expression that represents the volume.

- b. If we double the height, write an expression for the new height.
- c. Write an expression that represents the volume with the doubled height.
- d. Write an equivalent expression using the commutative and associative properties to show the volume is twice the original volume.

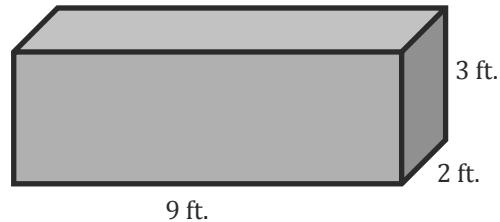
3. Use the cube to answer the following questions.

- a. Determine the volume of the cube.



- b. Determine the volume of a cube whose side lengths are half as long as the side lengths of the original cube.
- c. Determine the volume if the side lengths are one-fourth as long as the original cube's side lengths.
- d. Determine the volume if the side lengths are one-sixth as long as the original cube's side lengths.

- e. Explain the relationship between the side lengths and the volumes of the cubes.
4. Check to see if the relationship you found in Exercise 3 is the same for rectangular prisms.



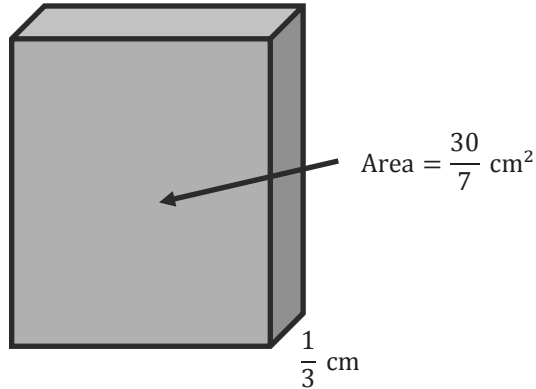
- a. Determine the volume of the rectangular prism.
- b. Determine the volume if all of the sides are half as long as the original lengths.
- c. Determine the volume if all of the sides are one-third as long as the original lengths.
- d. Is the relationship between the side lengths and the volume the same as the one that occurred in Exercise 3? Explain your answer.

5.

- a. If  $e$  represents a side length of the cube, create an expression that shows the volume of the cube.
- b. If we divide the side lengths by three, create an expression for the new side length.
- c. Write an expression that represents the volume of the cube with one-third the side length.
- d. Write an equivalent expression to show that the volume is  $\frac{1}{27}$  of the original volume.

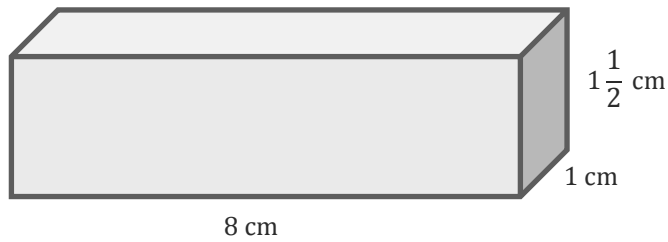
**Problem Set**

1. Determine the volume of the rectangular prism.



2. Determine the volume of the rectangular prism in Problem 1 if the height is quadrupled (multiplied by four). Then, determine the relationship between the volumes in Problem 1 and this prism.
3. The area of the base of a rectangular prism can be represented by  $B$ , and the height is represented by  $h$ .
- Write an equation that represents the volume of the prism.
  - If the area of the base is doubled, write an equation that represents the volume of the prism.
  - If the height of the prism is doubled, write an equation that represents the volume of the prism.
  - Compare the volume in parts (b) and (c). What do you notice about the volumes?
  - Write an expression for the volume of the prism if both the height and the area of the base are doubled.
4. Determine the volume of a cube with a side length of  $5\frac{1}{3}$  in.
5. Use the information in Problem 4 to answer the following:
- Determine the volume of the cube in Problem 4 if all of the side lengths are cut in half.
  - How could you determine the volume of the cube with the side lengths cut in half using the volume in Problem 4?

6. Use the rectangular prism to answer the following questions.



- a. Complete the table.

Length of Prism	Volume of Prism
$l = 8 \text{ cm}$	
$\frac{1}{2}l =$	
$\frac{1}{3}l =$	
$\frac{1}{4}l =$	
$2l =$	
$3l =$	
$4l =$	

- b. How did the volume change when the length was one-third as long?
- c. How did the volume change when the length was tripled?
- d. What conclusion can you make about the relationship between the volume and the length?
7. The sum of the volumes of two rectangular prisms, Box A and Box B, are  $14.325 \text{ cm}^3$ . Box A has a volume of  $5.61 \text{ cm}^3$ .
- a. Let  $B$  represent the volume of Box B in cubic centimeters. Write an equation that could be used to determine the volume of Box B.
- b. Solve the equation to determine the volume of Box B.
- c. If the area of the base of Box B is  $1.5 \text{ cm}^2$ , write an equation that could be used to determine the height of Box B. Let  $h$  represent the height of Box B in centimeters.
- d. Solve the equation to determine the height of Box B.

## Lesson 14: Volume in the Real World

### Classwork

#### Example 1

- a. The area of the base of a sandbox is  $9\frac{1}{2}$  ft<sup>2</sup>. The volume of the sandbox is  $7\frac{1}{8}$  ft<sup>3</sup>. Determine the height of the sandbox.

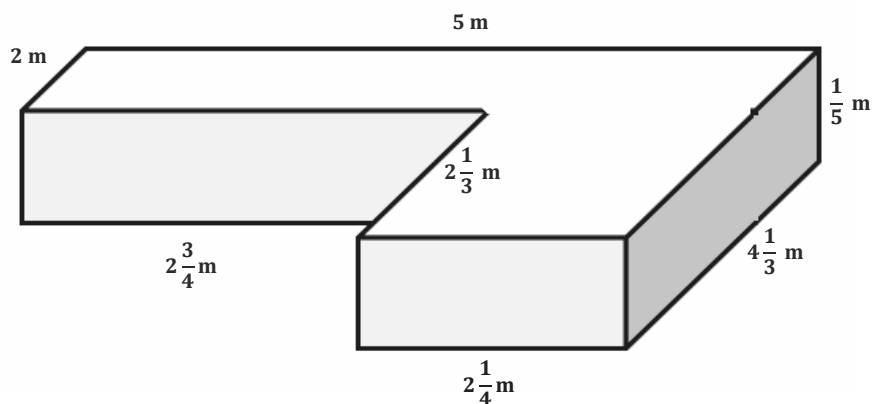


- b. The sandbox was filled with sand, but after the kids played, some of the sand spilled out. Now, the sand is at a height of  $\frac{1}{2}$  ft. Determine the volume of the sand in the sandbox after the children played in it.



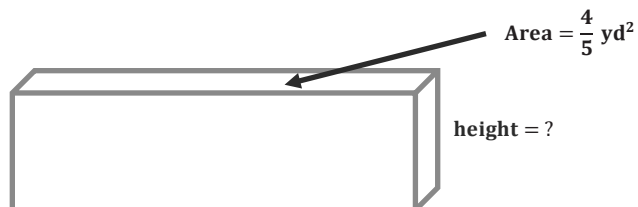
**Example 2**

A special-order sandbox has been created for children to use as an archeological digging area at the zoo. Determine the volume of the sandbox.

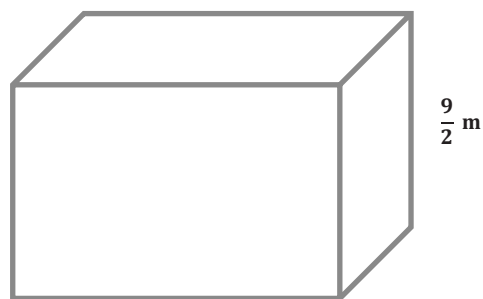
**Exercises**

1.

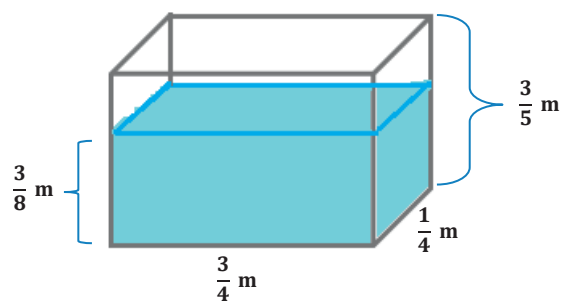
- a. The volume of the rectangular prism is  $\frac{35}{15} \text{ yd}^3$ . Determine the missing measurement using a one-step equation.



- b. The volume of the box is  $\frac{45}{6} \text{ m}^3$ . Determine the area of the base using a one-step equation.



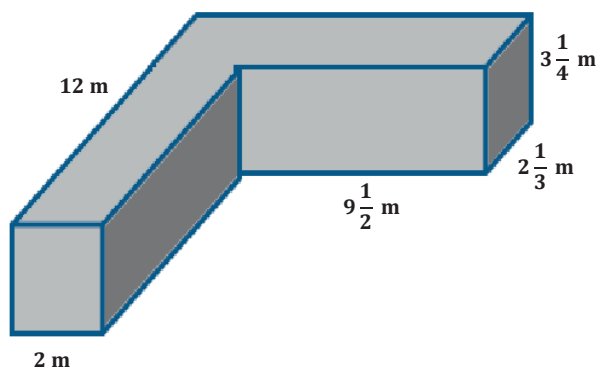
2. Marissa's fish tank needs to be filled with more water.
- a. Determine how much water the tank can hold.



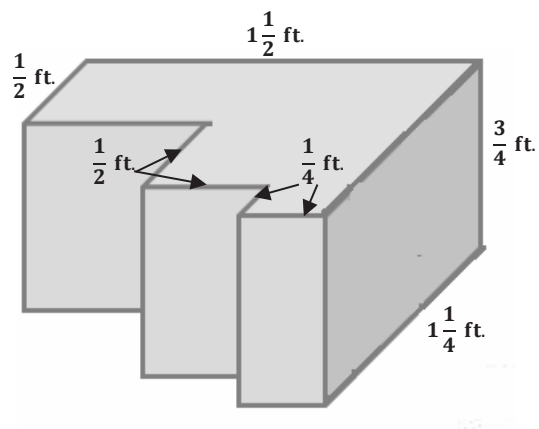
- b. Determine how much water is already in the tank.
- c. How much more water is needed to fill the tank?

3. Determine the volume of the composite figures.

a.

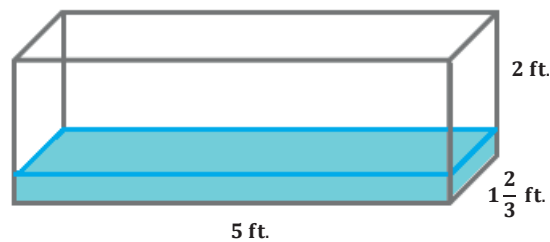


b.

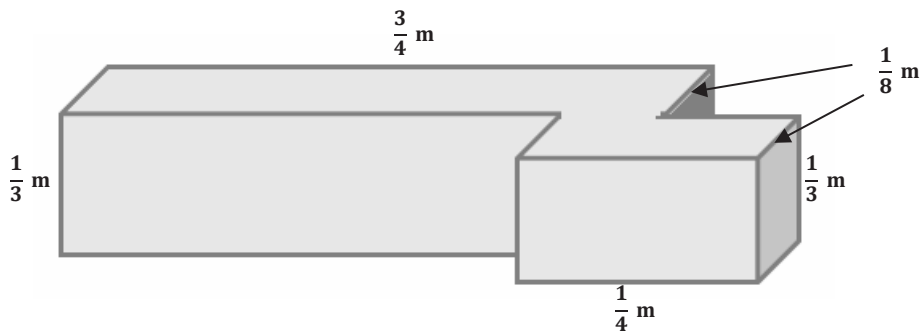


## Problem Set

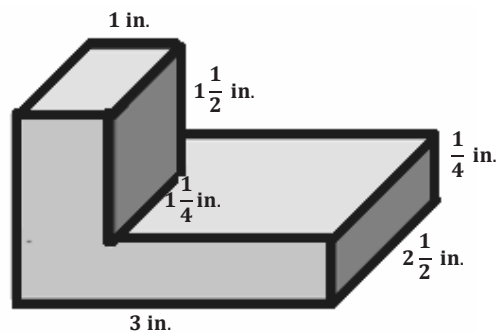
1. The volume of a rectangular prism is  $\frac{21}{12}$  ft<sup>3</sup>, and the height of the prism is  $\frac{3}{4}$  ft. Determine the area of the base.
2. The volume of a rectangular prism is  $\frac{10}{21}$  ft<sup>3</sup>. The area of the base is  $\frac{2}{3}$  ft<sup>2</sup>. Determine the height of the rectangular prism.
3. Determine the volume of the space in the tank that still needs to be filled with water if the water is  $\frac{1}{3}$  ft. deep.



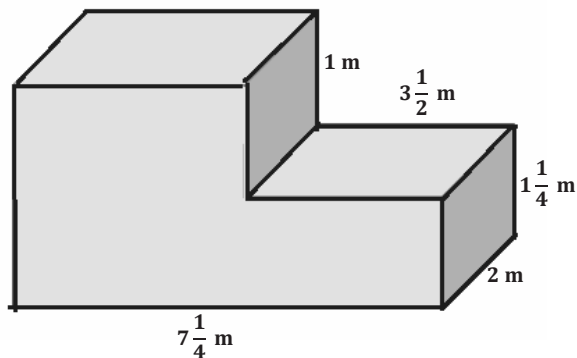
4. Determine the volume of the composite figure.



5. Determine the volume of the composite figure.



6.



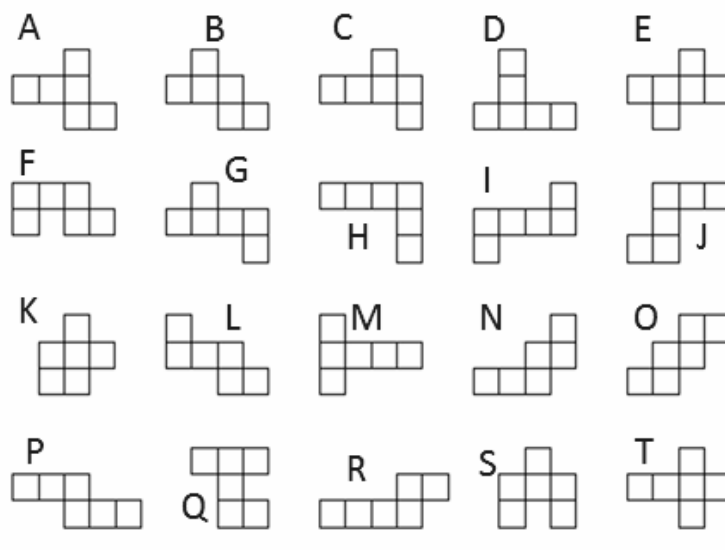
- Write an equation to represent the volume of the composite figure.
- Use your equation to calculate the volume of the composite figure.

## Lesson 15: Representing Three-Dimensional Figures Using Nets

### Classwork

#### Exercise: Cube

1. Nets are two-dimensional figures that can be folded into three-dimensional solids. Some of the drawings below are nets of a cube. Others are not cube nets; they can be folded, but not into a cube.



- Experiment with the larger cut-out patterns provided. Shade in each of the figures above that can fold into a cube.
- Write the letters of the figures that can be folded into a cube.
- Write the letters of the figures that cannot be folded into a cube.

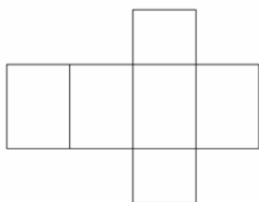
**Lesson Summary**

**NET:** If the surface of a 3-dimensional solid can be cut along sufficiently many edges so that the faces can be placed in one plane to form a connected figure, then the resulting system of faces is called a *net of the solid*.

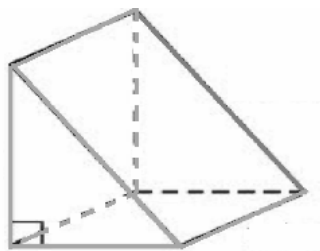
**Problem Set**

1. Match the following nets to the picture of its solid. Then, write the name of the solid.

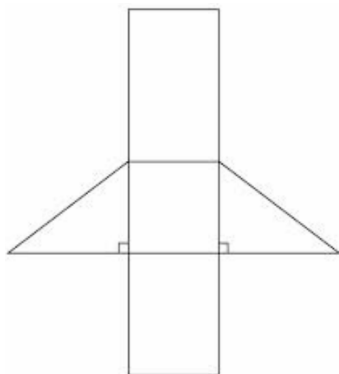
a.



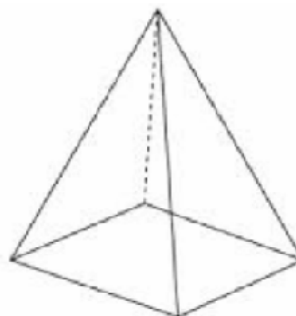
d.



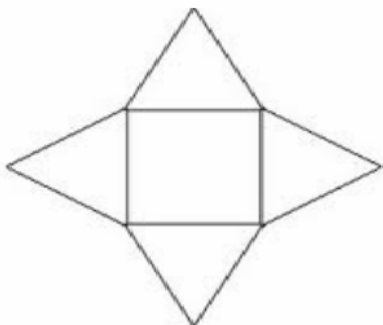
b.



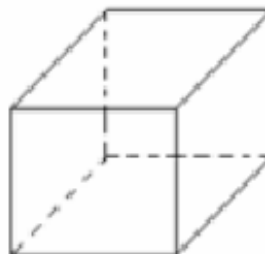
e.



c.

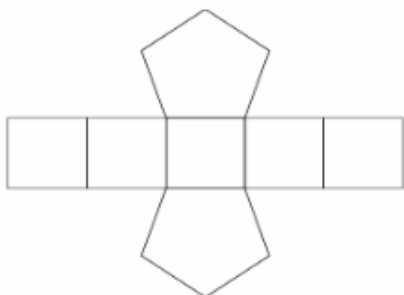


f.

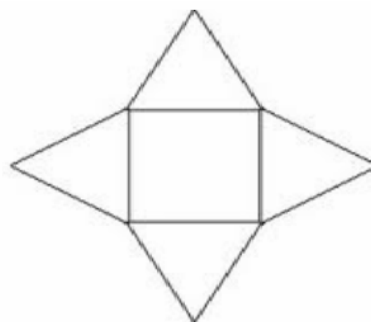


2. Sketch a net that can fold into a cube.
3. Below are the nets for a variety of prisms and pyramids. Classify the solids as prisms or pyramids, and identify the shape of the base(s). Then, write the name of the solid.

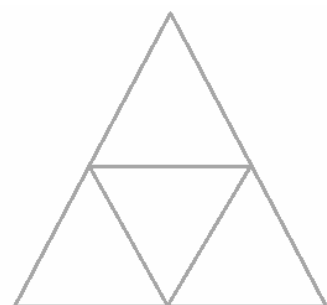
a.



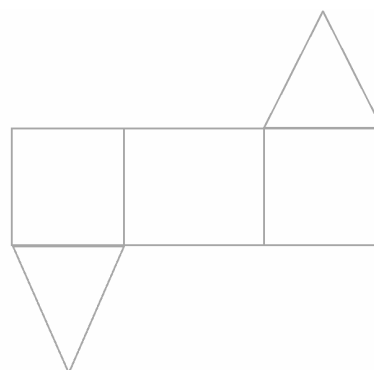
b.



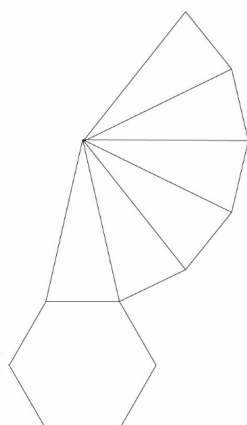
c.



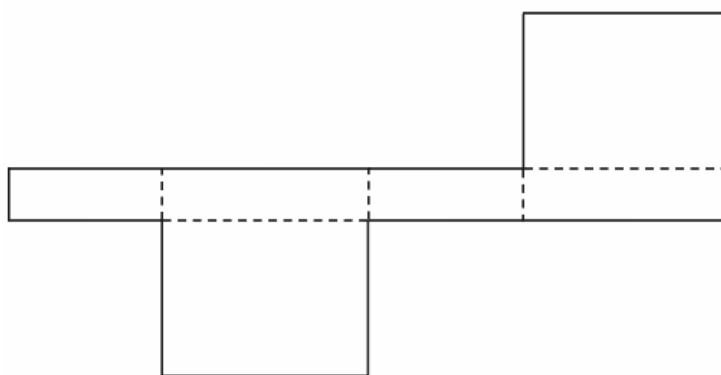
d.



e.



f.



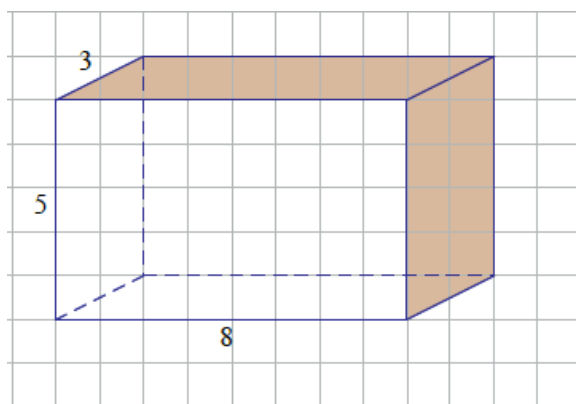


## Lesson 16: Constructing Nets

### Classwork

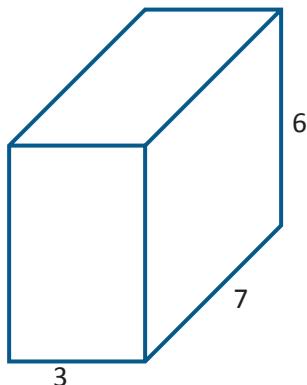
#### Opening Exercise

Sketch the faces in the area below. Label the dimensions.



**Exploratory Challenge 1: Rectangular Prisms**

- a. Use the measurements from the solid figures to cut and arrange the faces into a net. (Note: All measurements are in centimeters.)



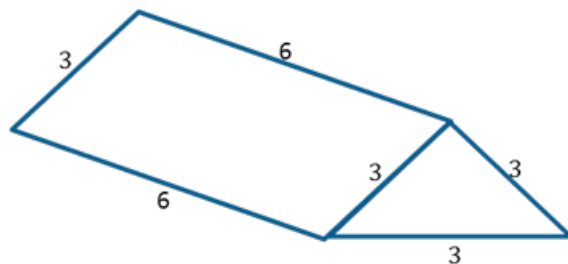
- b. A juice box measures 4 inches high, 3 inches long, and 2 inches wide. Cut and arrange all 6 faces into a net. (Note: All measurements are in inches.)



- c. Challenge: Write a numerical expression for the total area of the net for part (b). Explain each term in your expression.

**Exploratory Challenge 2: Triangular Prisms**

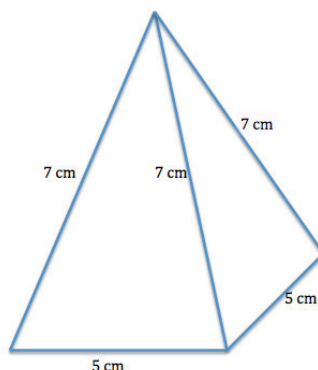
Use the measurements from the triangular prism to cut and arrange the faces into a net. (Note: All measurements are in inches.)



**Exploratory Challenge 3: Pyramids**

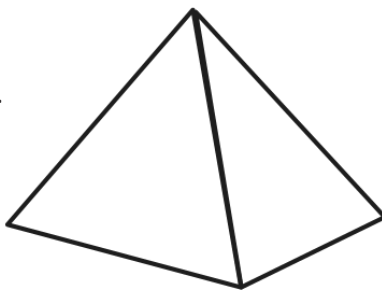
Pyramids are named for the shape of the base.

- a. Use the measurements from this square pyramid to cut and arrange the faces into a net. Test your net to be sure it folds into a square pyramid.



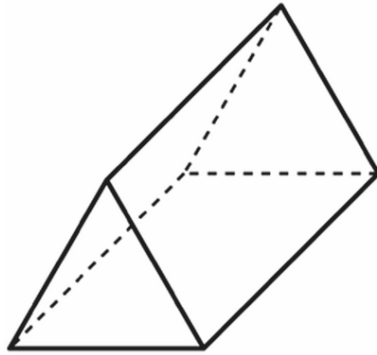
- b. A triangular pyramid that has equilateral triangles for faces is called a tetrahedron. Use the measurements from this tetrahedron to cut and arrange the faces into a net.

All edges are 4 in. in length.



**Problem Set**

1. Sketch and label the net of the following solid figures, and label the edge lengths.
  - a. A cereal box that measures 13 inches high, 7 inches long, and 2 inches wide
  - b. A cubic gift box that measures 8 cm on each edge
  - c. Challenge: Write a numerical expression for the total area of the net in part (b). Tell what each of the terms in your expression means.
2. This tent is shaped like a triangular prism. It has equilateral bases that measure 5 feet on each side. The tent is 8 feet long. Sketch the net of the tent, and label the edge lengths.



3. The base of a table is shaped like a square pyramid. The pyramid has equilateral faces that measure 25 inches on each side. The base is 25 inches long. Sketch the net of the table base, and label the edge lengths.
4. The roof of a shed is in the shape of a triangular prism. It has equilateral bases that measure 3 feet on each side. The length of the roof is 10 feet. Sketch the net of the roof, and label the edge lengths.

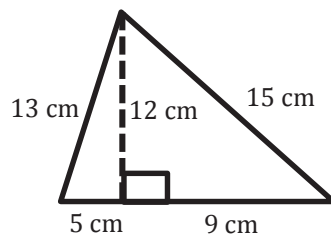
## Lesson 17: From Nets to Surface Area

### Classwork

#### Opening Exercise

- a. Write a numerical equation for the area of the figure below. Explain and identify different parts of the figure.

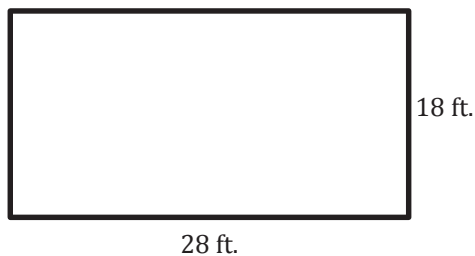
i.



- ii. How would you write an equation that shows the area of a triangle with base  $b$  and height  $h$ ?

- b. Write a numerical equation for the area of the figure below. Explain and identify different parts of the figure.

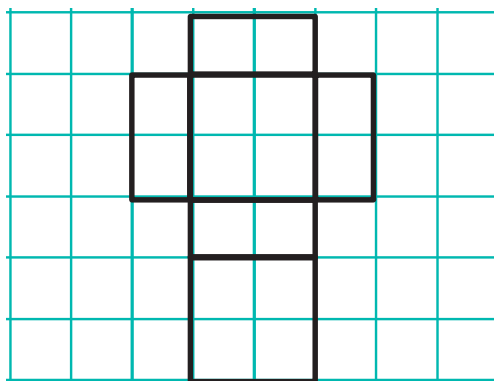
i.



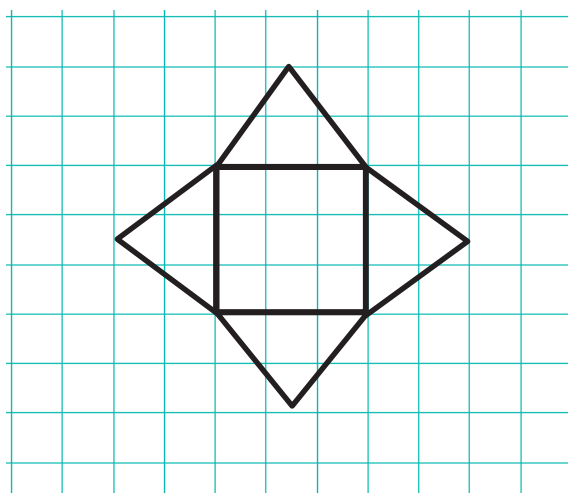
- ii. How would you write an equation that shows the area of a rectangle with base  $b$  and height  $h$ ?

**Example 1**

Use the net to calculate the surface area of the figure. (Note: all measurements are in centimeters.)

**Example 2**

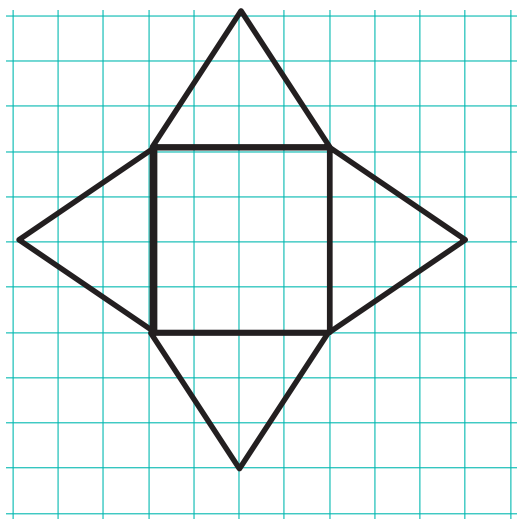
Use the net to write an expression for surface area. (Note: all measurements are in square feet.)



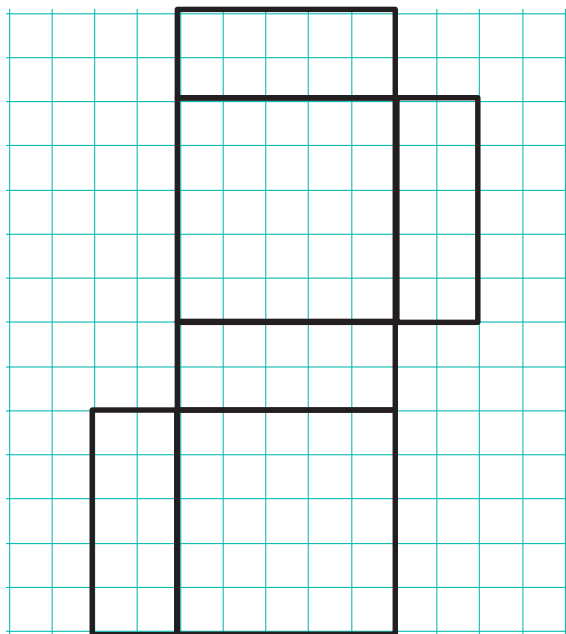
**Exercises**

Name the solid the net would create, and then write an expression for the surface area. Use the expression to determine the surface area. Assume that each box on the grid paper represents a  $1\text{ cm} \times 1\text{ cm}$  square. Explain how the expression represents the figure.

1.

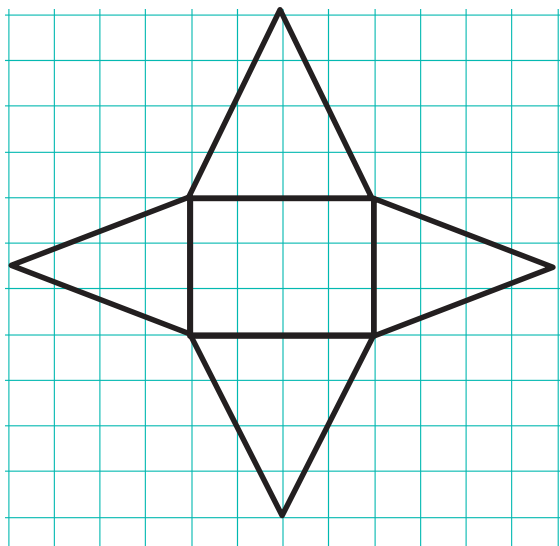


2.

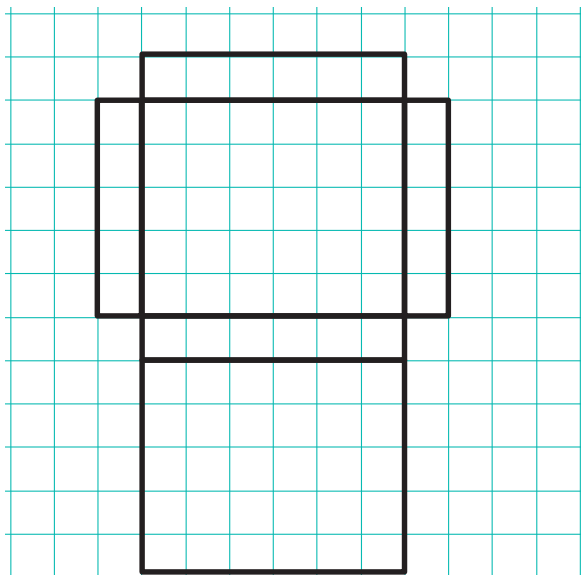




3.



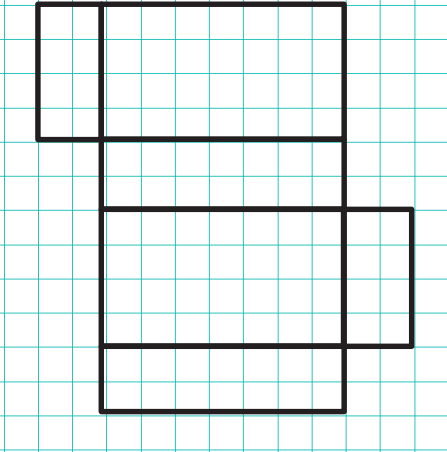
4.



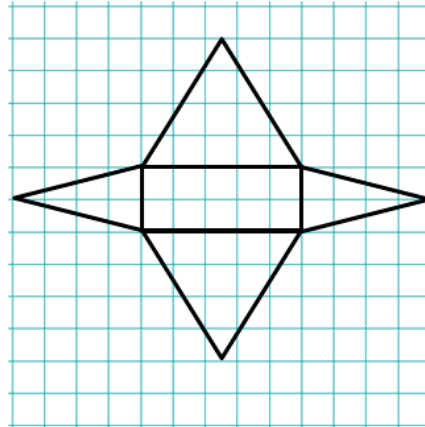
## Problem Set

Name the shape, and write an expression for surface area. Calculate the surface area of the figure. Assume each box on the grid paper represents a 1 ft.  $\times$  1 ft. square.

1.

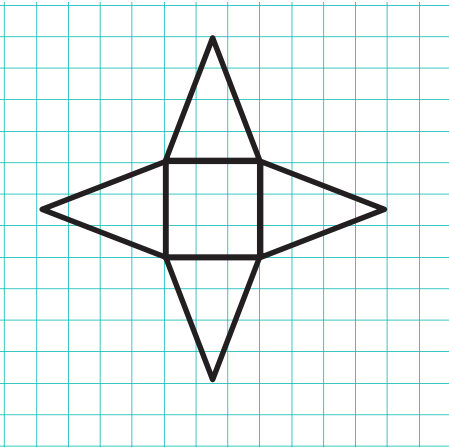


2.



Explain the error in each problem below. Assume each box on the grid paper represents a 1 m  $\times$  1 m square.

3.



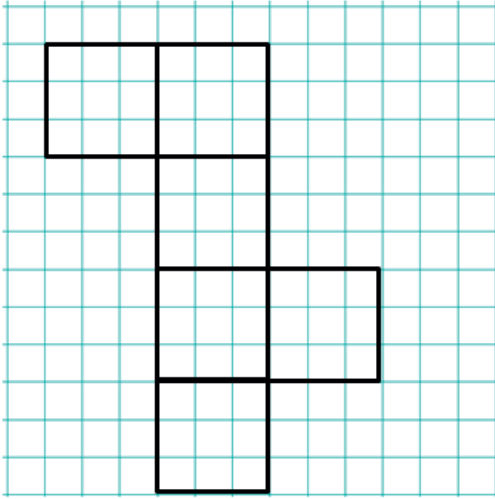
Name of Shape: Rectangular Pyramid, but more specifically a Square Pyramid

Area of Base:  $3\text{ m} \times 3\text{ m} = 9\text{ m}^2$

Area of Triangles:  $3\text{ m} \times 4\text{ m} = 12\text{ m}^2$

Surface Area:  $9\text{ m}^2 + 12\text{ m}^2 + 12\text{ m}^2 + 12\text{ m}^2 + 12\text{ m}^2 = 57\text{ m}^2$

4.



Name of Shape: Rectangular Prism or, more specifically, a Cube

Area of Faces:  $3\text{ m} \times 3\text{ m} = 9\text{ m}^2$ Surface Area:  $9\text{ m}^2 + 9\text{ m}^2 + 9\text{ m}^2 + 9\text{ m}^2 + 9\text{ m}^2 = 45\text{ m}^2$ 

5. Sofia and Ella are both writing expressions to calculate the surface area of a rectangular prism. However, they wrote different expressions.

- a. Examine the expressions below, and determine if they represent the same value. Explain why or why not.

Sofia's Expression:

$$(3\text{ cm} \times 4\text{ cm}) + (3\text{ cm} \times 4\text{ cm}) + (3\text{ cm} \times 5\text{ cm}) + (3\text{ cm} \times 5\text{ cm}) + (4\text{ cm} \times 5\text{ cm}) + (4\text{ cm} \times 5\text{ cm})$$

Ella's Expression:

$$2(3\text{ cm} \times 4\text{ cm}) + 2(3\text{ cm} \times 5\text{ cm}) + 2(4\text{ cm} \times 5\text{ cm})$$

- b. What fact about the surface area of a rectangular prism does Ella's expression show more clearly than Sofia's?

## Lesson 18: Determining Surface Area of Three-Dimensional Figures

### Classwork

#### Opening Exercise

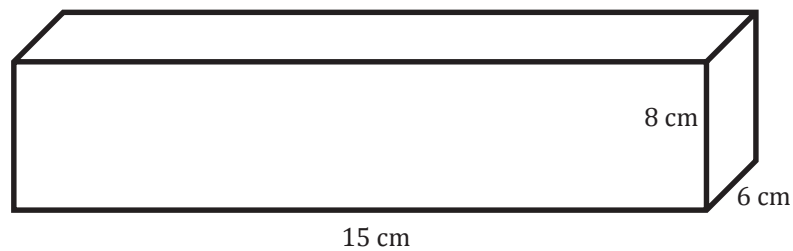
- a. What three-dimensional figure does the net create?
- b. Measure (in inches) and label each side of the figure.
- c. Calculate the area of each face, and record this value inside the corresponding rectangle.
- d. How did we compute the surface area of solid figures in previous lessons?
- e. Write an expression to show how we can calculate the surface area of the figure above.
- f. What does each part of the expression represent?
- g. What is the surface area of the figure?

**Example 1**

Fold the net used in the Opening Exercise to make a rectangular prism. Have the two faces with the largest area be the bases of the prism. Fill in the first row of the table below.

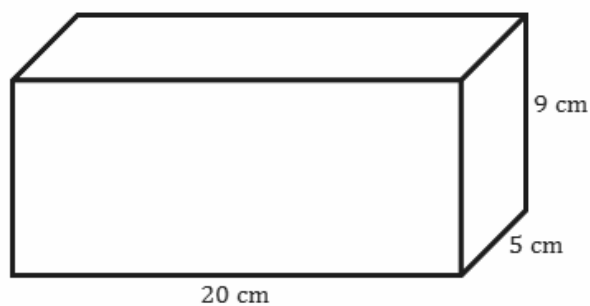
Area of Top (base)	Area of Bottom (base)	Area of Front	Area of Back	Area of Left Side	Area of Right Side

Examine the rectangular prism below. Complete the table.



Area of Top (base)	Area of Bottom (base)	Area of Front	Area of Back	Area of Left Side	Area of Right Side

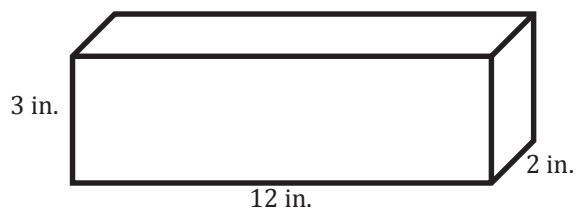
## Example 2



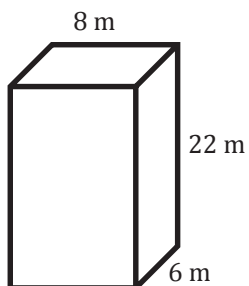
## Exercises 1–3

1. Calculate the surface area of each of the rectangular prisms below.

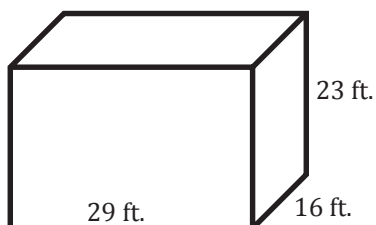
a.

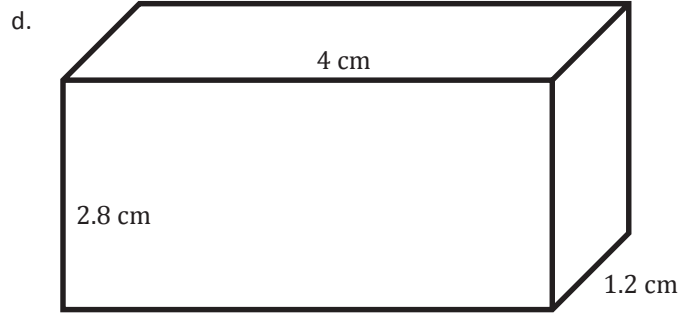


b.

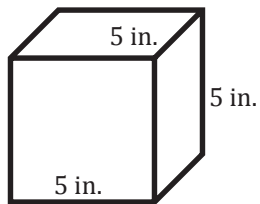


c.





2. Calculate the surface area of the cube.



3. All the edges of a cube have the same length. Tony claims that the formula  $SA = 6s^2$ , where  $s$  is the length of each side of the cube, can be used to calculate the surface area of a cube.
- Use the dimensions from the cube in Problem 2 to determine if Tony's formula is correct.
  - Why does this formula work for cubes?
  - Becca does not want to try to remember two formulas for surface area, so she is only going to remember the formula for a cube. Is this a good idea? Why or why not?

**Lesson Summary**

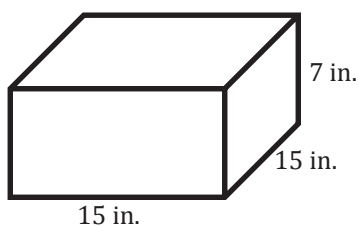
Surface Area Formula for a Rectangular Prism:  $SA = 2lw + 2lh + 2wh$

Surface Area Formula for a Cube:  $SA = 6s^2$

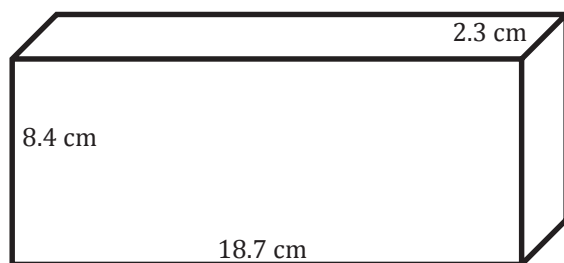
**Problem Set**

Calculate the surface area of each figure below. Figures are not drawn to scale.

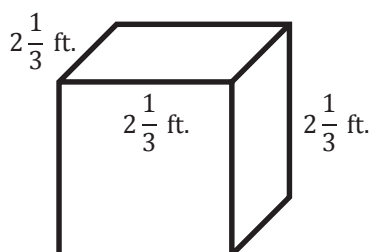
1.



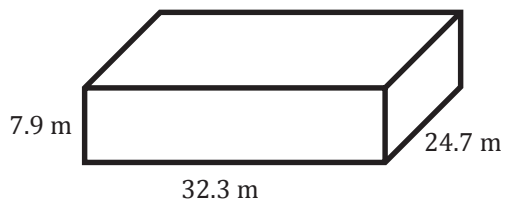
2.



3.

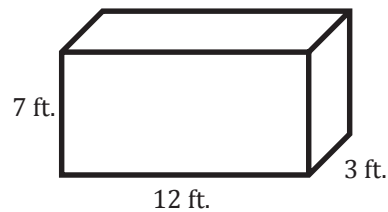


4.





5. Write a numerical expression to show how to calculate the surface area of the rectangular prism. Explain each part of the expression.



6. When Louie was calculating the surface area for Problem 4, he identified the following:

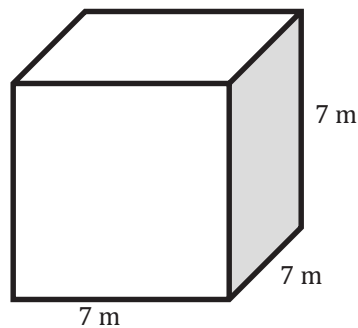
length = 24.7 m, width = 32.3 m, and height = 7.9 m.

However, when Rocko was calculating the surface area for the same problem, he identified the following:

length = 32.3 m, width = 24.7 m, and height = 7.9 m.

Would Louie and Rocko get the same answer? Why or why not?

7. Examine the figure below.



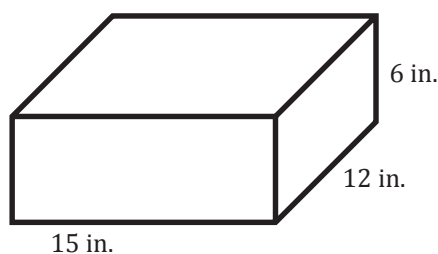
- What is the most specific name of the three-dimensional shape?
- Write two different expressions for the surface area.
- Explain how these two expressions are equivalent.

## Lesson 19: Surface Area and Volume in the Real World

### Classwork

#### Opening Exercise

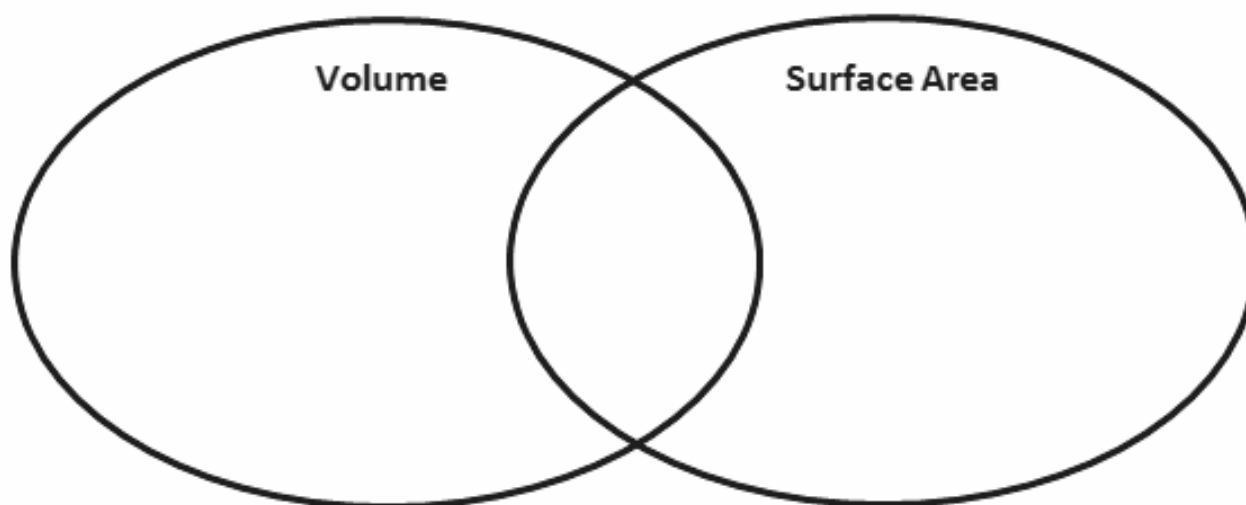
A box needs to be painted. How many square inches need to be painted to cover the entire surface of the box?



A juice box is 4 in. tall, 1 in. wide, and 2 in. long. How much juice fits inside the juice box?

How did you decide how to solve each problem?

#### Discussion



**Example 1**

Vincent put logs in the shape of a rectangular prism outside his house. However, it is supposed to snow, and Vincent wants to buy a cover so the logs stay dry. If the pile of logs creates a rectangular prism with these measurements:

33 cm long, 12 cm wide, and 48 cm high,

what is the minimum amount of material needed to cover the pile of logs?

**Exercises**

Use your knowledge of volume and surface area to answer each problem.

1. Quincy Place wants to add a pool to the neighborhood. When determining the budget, Quincy Place determined that it would also be able to install a baby pool that required less than 15 cubic feet of water. Quincy Place has three different models of a baby pool to choose from.

Choice One:  $5 \text{ ft.} \times 5 \text{ ft.} \times 1 \text{ ft.}$

Choice Two:  $4 \text{ ft.} \times 3 \text{ ft.} \times 1 \text{ ft.}$

Choice Three:  $4 \text{ ft.} \times 2 \text{ ft.} \times 2 \text{ ft.}$

Which of these choices is best for the baby pool? Why are the others not good choices?

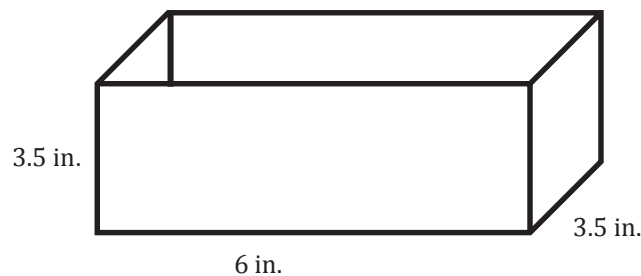
2. A packaging firm has been hired to create a box for baby blocks. The firm was hired because it could save money by creating a box using the least amount of material. The packaging firm knows that the volume of the box must be  $18 \text{ cm}^3$ .
- What are possible dimensions for the box if the volume must be exactly  $18 \text{ cm}^3$ ?
  - Which set of dimensions should the packaging firm choose in order to use the least amount of material? Explain.
3. A gift has the dimensions of  $50 \text{ cm} \times 35 \text{ cm} \times 5 \text{ cm}$ . You have wrapping paper with dimensions of  $75 \text{ cm} \times 60 \text{ cm}$ . Do you have enough wrapping paper to wrap the gift? Why or why not?
4. Tony bought a flat-rate box from the post office to send a gift to his mother for Mother's Day. The dimensions of the medium-size box are  $14 \text{ inches} \times 12 \text{ inches} \times 3.5 \text{ inches}$ . What is the volume of the largest gift he can send to his mother?

5. A cereal company wants to change the shape of its cereal box in order to attract the attention of shoppers. The original cereal box has dimensions of 8 inches  $\times$  3 inches  $\times$  11 inches. The new box the cereal company is thinking of would have dimensions of 10 inches  $\times$  10 inches  $\times$  3 inches.

a. Which box holds more cereal?

b. Which box requires more material to make?

6. Cinema theaters created a new popcorn box in the shape of a rectangular prism. The new popcorn box has a length of 6 inches, a width of 3.5 inches, and a height of 3.5 inches but does not include a lid.



a. How much material is needed to create the box?

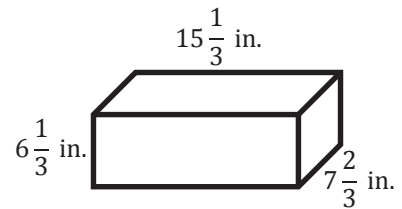
b. How much popcorn does the box hold?

**Problem Set**

Solve each problem below.

1. Dante built a wooden, cubic toy box for his son. Each side of the box measures 2 feet.
  - a. How many square feet of wood did he use to build the box?
  - b. How many cubic feet of toys will the box hold?
2. A company that manufactures gift boxes wants to know how many different-sized boxes having a volume of 50 cubic centimeters it can make if the dimensions must be whole centimeters.
  - a. List all the possible whole number dimensions for the box.
  - b. Which possibility requires the least amount of material to make?
  - c. Which box would you recommend the company use? Why?

3. A rectangular box of rice is shown below. What is the greatest amount of rice, in cubic inches, that the box can hold?



4. The Mars Cereal Company has two different cereal boxes for Mars Cereal. The large box is 8 inches wide, 11 inches high, and 3 inches deep. The small box is 6 inches wide, 10 inches high, and 2.5 inches deep.
  - a. How much more cardboard is needed to make the large box than the small box?
  - b. How much more cereal does the large box hold than the small box?
5. A swimming pool is 8 meters long, 6 meters wide, and 2 meters deep. The water-resistant paint needed for the pool costs \$6 per square meter. How much will it cost to paint the pool?
  - a. How many faces of the pool do you have to paint?
  - b. How much paint (in square meters) do you need to paint the pool?
  - c. How much will it cost to paint the pool?
6. Sam is in charge of filling a rectangular hole with cement. The hole is 9 feet long, 3 feet wide, and 2 feet deep. How much cement will Sam need?

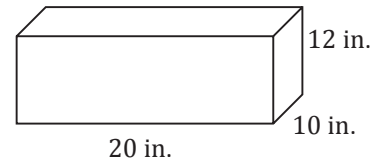
7. The volume of Box D subtracted from the volume of Box C is 23.14 cubic centimeters. Box D has a volume of 10.115 cubic centimeters.
- Let  $C$  be the volume of Box C in cubic centimeters. Write an equation that could be used to determine the volume of Box C.
  - Solve the equation to determine the volume of Box C.
  - The volume of Box C is one-tenth the volume another box, Box E. Let  $E$  represent the volume of Box E in cubic centimeters. Write an equation that could be used to determine the volume of Box E, using the result from part (b).
  - Solve the equation to determine the volume of Box E.

## Lesson 19a: Applying Surface Area and Volume to Aquariums

### Classwork

#### Opening Exercise

Determine the volume of this aquarium.



#### Mathematical Modeling Exercise: Using Ratios and Unit Rate to Determine Volume

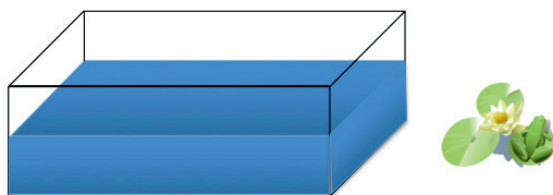
For his environmental science project, Jamie is creating habitats for various wildlife including fish, aquatic turtles, and aquatic frogs. For each of these habitats, he uses a standard aquarium with length, width, and height dimensions measured in inches, identical to the aquarium mentioned in the Opening Exercise. To begin his project, Jamie needs to determine the volume, or cubic inches, of water that can fill the aquarium.

Use the table below to determine the unit rate of gallons/cubic inches.

Gallons	Cubic Inches
1	
2	462
3	693
4	924
5	1,155

Determine the volume of the aquarium.



**Exercise 1**

- a. Determine the volume of the tank when filled with 7 gallons of water.
  
  
  
  
  
  
  
  
  
  
- b. Work with your group to determine the height of the water when Jamie places 7 gallons of water in the aquarium.

**Exercise 2**

- a. Use the table from Example 1 to determine the volume of the aquarium when Jamie pours 3 gallons of water into the tank.
  
  
  
  
  
  
  
  
  
  
- b. Use the volume formula to determine the missing height dimension.

## Exercise 3

- a. Using the table of values below, determine the unit rate of liters to gallon.

Gallons	Liters
1	
2	7.57
4	15.14

- b. Using this conversion, determine the number of liters needed to fill the 10-gallon tank.

- c. The ratio of the number of centimeters to the number of inches is 2.54: 1. What is the unit rate?

- d. Using this information, complete the table to convert the heights of the water in inches to the heights of the water in centimeters Jamie will need for his project at home.

Height (in inches)	Convert to Centimeters	Height (in centimeters)
1	$2.54 \frac{\text{centimeters}}{\text{inch}} \times 1 \text{ inch}$	2.54
3.465		
8.085		
11.55		

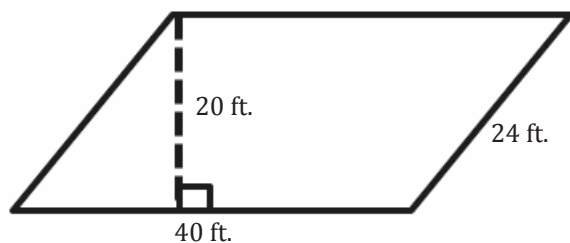
### Exercise 4

- a. Determine the amount of plastic film the manufacturer uses to cover the aquarium faces. Draw a sketch of the aquarium to assist in your calculations. Remember that the actual height of the aquarium is 12 inches.
- b. We do not include the measurement of the top of the aquarium since it is open without glass and does not need to be covered with film. Determine the area of the top of the aquarium, and find the amount of film the manufacturer uses to cover only the sides, front, back, and bottom.
- c. Since Jamie needs three aquariums, determine the total surface area of the three aquariums.

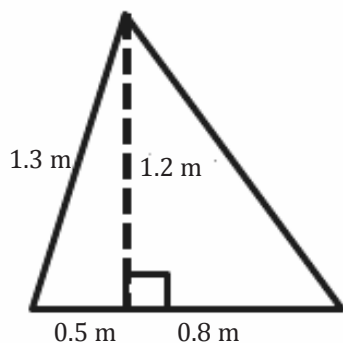
**Problem Set**

This Problem Set is a culmination of skills learned in this module. Note that the figures are not drawn to scale.

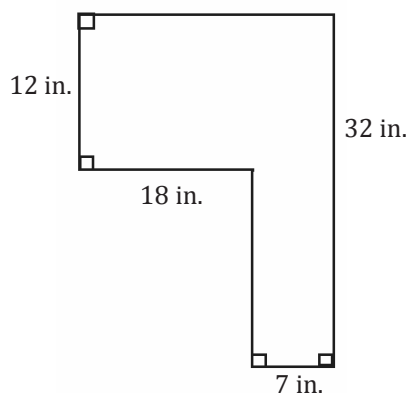
1. Calculate the area of the figure below.



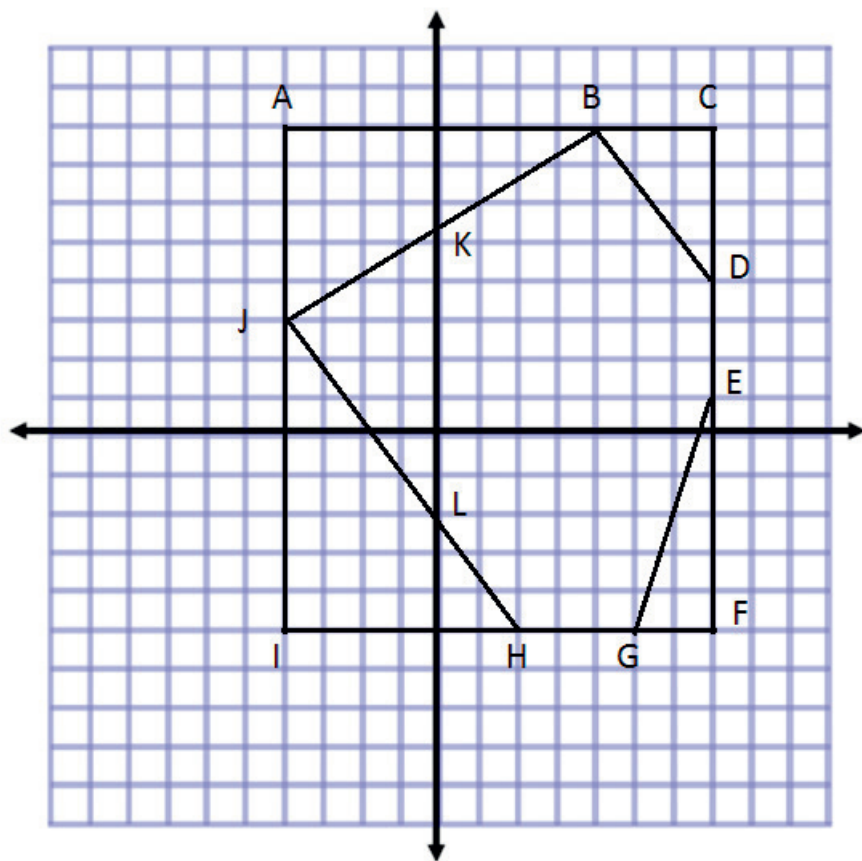
2. Calculate the area of the figure below.



3. Calculate the area of the figure below.



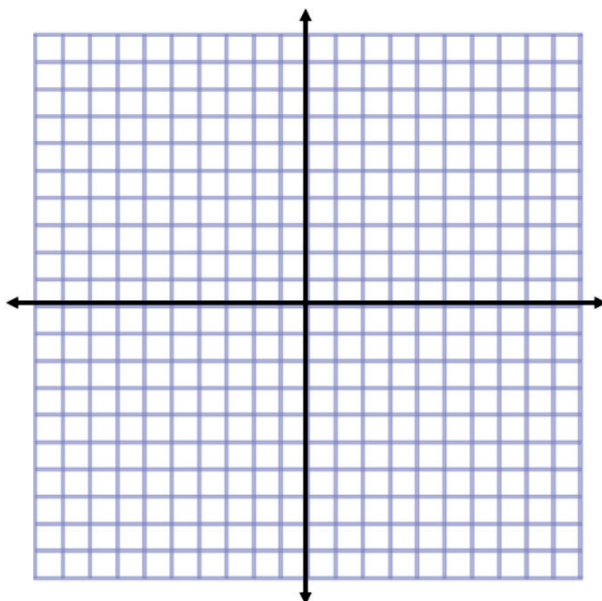
4. Complete the table using the diagram on the coordinate plane.



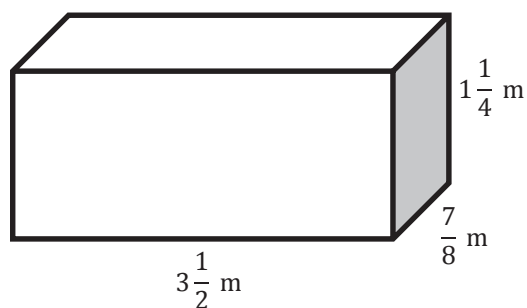
Line Segment	Point	Point	Distance	Proof
$\overline{AB}$				
$\overline{CE}$				
$\overline{GI}$				
$\overline{HI}$				
$\overline{IJ}$				
$\overline{AI}$				
$\overline{AJ}$				

5. Plot the points below, and draw the shape. Then, determine the area of the polygon.

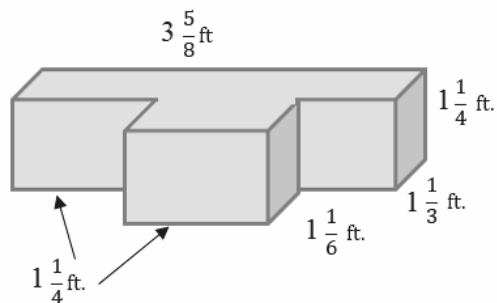
$A(-3, 5)$ ,  $B(4, 3)$ ,  $C(0, -5)$



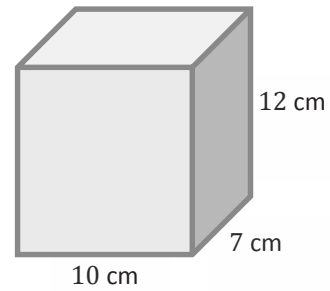
6. Determine the volume of the figure.



7. Give at least three more expressions that could be used to determine the volume of the figure in Problem 6.
8. Determine the volume of the irregular figure.



9. Draw and label a net for the following figure. Then, use the net to determine the surface area of the figure.



10. Determine the surface area of the figure in Problem 9 using the formula  $SA = 2lw + 2lh + 2wh$ . Then, compare your answer to the solution in Problem 9.
11. A parallelogram has a base of 4.5 cm and an area of  $9.495 \text{ cm}^2$ . Tania wrote the equation  $4.5x = 9.495$  to represent this situation.
- Explain what  $x$  represents in the equation.
  - Solve the equation for  $x$  and determine the height of the parallelogram.
12. Triangle A has an area equal to one-third the area of Triangle B. Triangle A has an area of  $3\frac{1}{2}$  square meters.
- Gerard wrote the equation  $\frac{B}{3} = 3\frac{1}{2}$ . Explain what  $B$  represents in the equation.
  - Determine the area of Triangle B.